KINESTATIC ANALYSIS OF MULTI-FINGERED HANDS

BY

LOTFI ROMDHANE

A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT
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TO

MY PARENTS

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KAOUTHAR

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Lotfi Romdhane

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Chairman: Dr. Joseph Duffy
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This work is concerned with the handling of an object by an articulated multi-fingered hand. By handling, we mean the secure grasp of a general object plus the manipulation of this object by the fingers without motion of the arm on which the hand is mounted. Hypothetical joints are used to express the freedom at the contact of the object with each fingertip. A pair of matrices are introduced that relate the small motions of the joints to the relative displacement of the object and the fingertips, and are derived using reciprocal screws.

This analysis permits not only the determination of the necessary conditions for a secure handling of the object, but also the investigation of the special configurations that may occur. In the case of a grasp with point contact with friction, a very important result is derived which leads to a considerable simplification in the analysis of these

matrices. Further, it is shown that there exists a particular point, called the center of the grasp, at which the direction of the force applied to the object does not influence the contact intensities. Following this, it is demonstrated that the vector containing the contact intensities, generated by a unit couple applied to the object, generates an ellipsoid, which is defined as the grasp ellipsoid. The two smaller axes of this ellipsoid show where to place the contacts to minimize the contact forces for a given task. This result is a very important one because it yields a quality index that can be used as a criterion in the choice between different proposed grasps.

Lastly, a computer software is developed to draw a solid model of the hand and to animate it. Two modes are available for the user. In the first one, the user chooses the initial and final position and orientation of the grasped object and a straight-line path is generated and displayed on the screen. In the second one, the user leads the object to its goal position using the keyboard as a teach-pendent. The path can then be previewed and saved in a file for later use.

CHAPTER 1
INTRODUCTION

Simple two-finger gripper designs that are commonplace in industry rely on friction to restrain the grasped object. Examples include parallel finger gripper (Figure 1.1) and pivoting finger. gripper (Figure 1.2). For oddly shaped objects complete restriction might not be possible using the above designs. Inflatables grippers and snakelike fingers are used in some cases to accomodate objects with intricate shapes. Chen [1982] shows several gripping mechanisms for industrial robots.

Several difficulties arise when these kinds of hands are applied to flexible manufacturing systems. These difficulties prevent industrial robots and their grippers from accurately grasping and manipulating objects. For instance, these simple gripper designs are not well adapted to a wide range of object shapes, and industrial manipulators to which the grippers are attached are usually not well suited for both gross and fine motion.

The modeling of the human hand is a formidable task. The human hand is a sophisticated mechanical system of muscles and ligaments and bones with at least 18 joint articulations with 22 degrees of freedom or more. Nonetheless, the need for more versatile end-effectors in robotics has led more and more researchers to design mechanical hands (Crossley & Umholtz [1977], Skinner [1975], Okada[1979], Salisbury [1982], and Jacobson et al. [1984]).

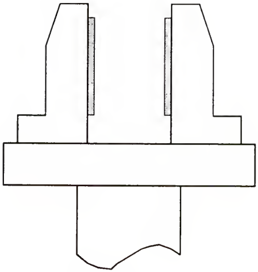


Figure 1.1

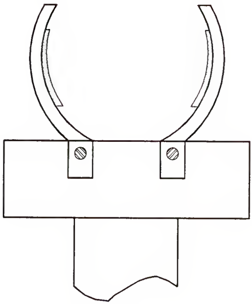


Figure 1.2

Recently, there has been a growing interest in studying multi-fingered hands capable of manipulating the grasped object, see for example Salisbury [1982], Kerr [1984], Kerr and Roth [1986], Kobayashi [1985], Holzman and McCarthy [1985], Kerr and Sangar [1986], Hanafusa et al. [1983], Cutkosky [1985], Nguyen [1987a, b], and Ji and Roth [1987]. In the work done by Salisbury [1982], which was subsequently extended by Kerr [1984] and Kobayashi [1985], a pair of matrices describing the kinestatics (instantaneous motions and statics) of the hand were presented. Kerr and Sanger [1986] introduced a stiffness matrix for a grasp with elastic contacts. Following this they deduced the external force on the grasped object and determined whether it will slip. When slipping occurs the resulting motion is determined.

In this work the matrices derived by Maher [1983] and Maher and Duffy [1984] for fully parallel devices using reciprocal screws are modified and applied to the grasping problem. Whilst these matrices are similar to those due to Salisbury, the method of solution for the grasping problem is novel and avoids the use of the generalized inverse (Salisbury [1982], and Kerr [1984]) or projective matrix techniques (Kobayashi [1985]) which lead to non-invariant results with respect to translation of origin and also non-invariance with respect to a change of scale (see Rico [1988], Griffis [1988], Lipkin [1985], and Lipkin and Duffy [1985, 1986]).

Briefly, noninvariance occurs because one of the matrices, which will be referred to as the J matrix, is necessarily generated by a dual vector formulation rather than by the classical matrix formulation. Transformations incorporating the orthogonal product of dual vectors have no physical meaning simply because the first and last three components of these vectors have different dimensions. Such transformations cannot be used since the results are mathematically meaningless.

In order to resolve these problems, an invariant modified singular value decomposition (MSVD) of this matrix is presented in Appendix B. Essentially, this new method combines the singular value decomposition (SVD) (Steven [1980] and Beni Israel et al. [1974]) with the dual operator (Lipkin [1985]). Using this procedure an analysis of the characteristic matrix equation of the hand revealed the different special configurations that might occur during motion. Furthermore, this novel method yields, in the case of point contact with friction, a very simple yet powerful means of analyzing these matrices. The analysis of the $m \times n$ dual matrix ($m,n > 6$) reduces to the analysis of a simple 3×3 matrix, which simplifies the computations tremendously. Also the invariance of this method with respect to a change of origin has been proven. Following this, a force analysis using the principle of virtual work together with a characteristic matrix equation yields an equation relating the external wrench on the object to the joint torques. It is important to recognize that the magnitude (or norm) of a screw has no physical meaning because the first three and last three components of the dual vector have different dimensions.

A summary of each chapter is now presented.

The objective of chapter 2 is to present the necessary tools for the analysis of the multi-fingered hands. Screw theory forms the basis of this work. A comprehensive theory of screws and reciprocal screw systems was first established by Ball [1900] and a modern presentation with application to mechanisms and robot manipulators is presented by Hunt [1978]. It is well established that a general system of forces and couples acting upon a rigid body can be reduced to a wrench acting along a screw and that the instantaneous motion of a rigid body is equivalent to a twist on a screw. Screw theory has been widely applied to the analysis of serial manipulators (Hunt [1987], Duffy [1980, 1983], and Roth

and Bottema [1979]), but few (Maher [1983], Salisbury [1982], and Sugimoto [1986, 1988]) were interested in parallel mechanisms.

In chapter 3 a kinematic analysis of multi-fingered hands is presented. Using the results of the previous chapter concerning serial manipulators and the work of Maher [1983], we extended the analysis to multi-fingered hands. A characteristic matrix equation, $J^T \Delta \underline{\$} = K\underline{\Omega}$, governing the relation between the twist of the object, $\underline{\$}$, and the instantaneous joint velocities, $\underline{\Omega}$, was derived. The matrix J corresponds to the set of wrenches of constraints on the object, imposed by each contacting finger. Using the MSVD, developed in Appendix B, we solved the characteristic matrix equation.

First, a forward analysis yielding the twist of the object as a function of the joint angle velocities is carried out. This method uses the MSVD of the dual matrix J which uses the reciprocal product as the only product allowed between screws. This method leads then to invariant results that are physically meaningful. A reverse analysis of the multi-fingered hands is accomplished by inverting the matrix K. However, if the latter matrix is not square, the well known singular value decomposition can be applied to solve for $\underline{\Omega}$. A force analysis using the principle of virtual work together with the characteristic matrix equation, yields an equation relating the external wrench on the object to the joint torques. This analysis also uses the MSVD of the dual matrix J. A set of conditions securing the grasp were also derived.

Generally, for a single open serial chain of links, the special configurations occur when the Jacobian matrix becomes singular. However, in the case of a multi-fingered hands special configurations occur when the matrices J and/or K are no longer fully ranked. In chapter 4 we present the following four different cases that may arise:

- Case 1: Rank of J is less than 6
- Case 2: The matrix K is not fully ranked
- Case 3: K and J are not fully ranked
- Case 4: The matrix $K = 0$

Once a multi-fingered hand has been built, one of the most basic questions to be answered is the determination of the best grasp. This subject is treated in chapter 5. Namely, given an object together with a task to perform, what is the appropriate grasp of the object so that the task can be executed efficiently? Obviously different criteria yield different results, and the following cases are examined,

- Minimization of the joint angular velocities for a given unit twist of the object.
- Minimization of the joint torques (or the contact intensities) for a given unit wrench applied to the object.
- Minimization of the contact intensities or angular velocities for a given grasp configuration. In the case of point contact with friction, the concepts of grasp ellipsoid and the center of the grasp are introduced. They prove to be powerful tools to help the search for an optimal grasp.

In chapter 6, a computer graphics simulation for a 3-fingered hand is developed. All the theory developed in the previous chapters is incorporated in this software, and it provides a realistic animated solid model of the hand. An algorithm generating a straight line path, used together with a teaching mode, yields a spatial trajectory of the grasped object. Any selected motion is then previewed on the screen.

Chapter 7 contains a summary of this work, and some recommendations for future work.

CHAPTER 2
INTRODUCTION TO THE THEORY OF SCREWS

2.1 The Theory of Screws

The screw theory is an elegant and powerful method to investigate the kinestatics (kinematic and static) of serial and parallel manipulators. In this chapter, an overview of this theory and its application to the analysis of robot manipulators are presented. First, screws and screw systems are investigated. Then, two important screw operations, namely the reciprocal and orthogonal products, are presented.

2.1.1 Free Vectors and Line Vectors

An ordinary three-dimensional vector has magnitude and direction, but a vector may also have positional properties. In particular, two kinds of vectors occur in rigid body kinestatics: line vectors and free vectors.

A line vector, denoted by $\underline{\$}$, has magnitude and direction together with a location in space. The location of the line can be represented by its moment with respect to the origin. Let $\underline{S}$ be 3×1 vector representing the direction of $\underline{\$}$ and $\underline{r}$ a vector pointing from the origin to a point on the line. The moment of the line with respect to the origin is given by the vector $\underline{S}_o$

$$\underline{S}_o = \underline{r} \times \underline{S} \tag{2.1}$$

While $\underline{S}_o$ is origin dependent, the vector $\underline{S}$ is not. The six-dimensional vector $[\underline{S}, \underline{S}_o]^T$

completely describes the line. Although six numbers are specified to describe a unit line vector, only four are independently chosen due to the following constraints

$$\underline{S}^T \underline{S}_o = 0 \tag{2.2}$$

$$\|\underline{S}\| = 1 \tag{2.3}$$

The vectors $\underline{S}$ and $\underline{S}_o$ can be combined into a dual vector

$$\underline{\$} = \underline{S} + \epsilon \underline{S}_o \tag{2.4}$$

where $\epsilon^2 = \epsilon^3 = \ldots = 0$

or simply

$$\underline{\$} = \begin{bmatrix} \underline{S} \\ \underline{S}_o \end{bmatrix} = \begin{bmatrix} \underline{S} \\ \underline{r} \times \underline{S} \end{bmatrix} \tag{2.5}$$

A free vector with a magnitude and a direction given by a vector $\underline{S}$ is adequately represented by $\underline{S}$. We can put free vectors of the same algebraic footing as line vectors by expressing them in the form

$$\underline{\$} = \begin{bmatrix} \underline{0} \\ \underline{S} \end{bmatrix} \tag{2.6}$$

For a given reference frame the cartesian coordinate of the vectors $\underline{S}$ and $\underline{S}_o$ can be

written as follows

$$\underline{S} = \begin{bmatrix} L \\ M \\ N \end{bmatrix}, \text{ and } \underline{S}_o = \underline{r}\times\underline{S} = \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \tag{2.7}$$

Equation (2.5) becomes

$$\underline{\$} = \begin{bmatrix} L \\ M \\ N \\ P \\ Q \\ R \end{bmatrix} \tag{2.8}$$

The quantities L, M, N, P, Q, and R are called the Plücker coordinates of the line. The conditions given by Equations (2.2) and (2.3) become

$$LP + MQ + NR = 0 \tag{2.9}$$

$$L^2 + M^2 + N^2 = 1 \tag{2.10}$$

The free vector, Equation (2.6) can be written as

$$\underline{\$} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ L \\ M \\ N \end{bmatrix} \tag{2.11}$$

2.1.2 Screws and Screw Coordinates

A screw as defined by Ball [1900] is a directed line with which a linear magnitude, called the pitch, h, is associated. A screw is essentially a geometrical concept. Any finite or infinitesimal displacement of a rigid body can be represented in terms of a twist about a screw. Similarly, any system of forces and couples acting on a rigid body can be replaced by a single wrench on a screw (a force and a parallel couple).

The six Plücker line coordinates can be extended to a set of screw coordinates. If (L, M, N; P, Q, R) are the Plücker coordinates. of the screw axis then the screw coordinates, considering a pitch h, are (L, M, N; P*, Q*, R*) given by

$$\begin{bmatrix} L \\ M \\ N \\ P^* \\ Q^* \\ R^* \end{bmatrix} = \begin{bmatrix} L \\ M \\ N \\ P+hL \\ Q+hM \\ R+hN \end{bmatrix} \tag{2.12}$$

The above equation can be written also as

$$\underline{\$} = \begin{bmatrix} \underline{S} \\ \underline{S}_o \end{bmatrix} = \begin{bmatrix} \underline{S} \\ \underline{r}\times\underline{S} + h\underline{S} \end{bmatrix} \tag{2.13}$$

and
$$h = \frac{\underline{S}^T\underline{S}_o}{\underline{S}^T\underline{S}} = \frac{LP^*+MQ^*+NR^*}{L^2+M^2+N^2} \tag{2.14}$$

If the pitch, h, is zero then the screw coordinates are identical to the line coordinates. For a screw with infinite pitch, the direction vector vanishes, $\underline{S}=\underline{0}$ and the screw coordinates are identical to those for a free vector.

2.1.3 Screw Systems

A screw system is a subspace of a six-dimensional vector space over the real numbers. A screw is represented as a 6×1 column vector of the components in a standard basis, denoted by $\underline{e}_i$, which comprises three unit line vectors $\{\underline{e}_1, \underline{e}_2, \underline{e}_3\}$ forming the axes of the cartesian coordinate frame and three unit free vectors $\{\underline{e}_4, \underline{e}_5, \underline{e}_6\}$ parallel to the coordinate axes. Any screw $\underline{\$}$ may be expressed as

$$\underline{\$} = \sum_{i=1}^{6} x_i\underline{e}_i \tag{2.15}$$

where x_i are the canonical components of $\underline{\$}$.

Let us denote by Instantaneous Screw Axis (ISA) the set of unit line vectors and unit free vectors. All possible linear combinations of r ($r \leq 6$) independent ISA's form a screw system of order r. The order of the screw system of a joint is its connectivity, which is the number of relative degrees of freedom of the bodies connected by the joint. Screw systems associated with revolute, prismatic, and screw joints are of order one. The

screw system of order six contains the set of all ISA's which represent the six degrees of freedom of an unconstrained body in space.

2.1.4 Algebra of Screws

The addition and multiplication by a scalar of screws obey the rules of classical linear algebra. In this case, it is adequate to consider a screw as an element of $\mathbb{R}^6$. Also it can be proven that the six-system of screws is a vector space (Sugimoto & Duffy [1982]).

On the other hand, one can be tempted to use the classical definition of norm of a screw or the inner product of screws, but these latter concepts seem to be physically meaningless (see Lipkin [1985], Lipkin & Duffy [1986]). The reason for this is the fact that the first three and last three components of a screw are of different dimension. Also, the vector $\underline{S}_o$ is origin dependent, leading to a different magnitude of the same screw when a change of origin is performed.

Another concept, which is a direct consequence of the above reasoning, is the orthogonality of screws. Two screws $\underline{V}_1$ and $\underline{V}_2$ are said to be orthogonal if their inner product, defined as $\underline{V}_1^T\underline{V}_2$, is equal to zero. In the following example we will show that the property between the same two screws is affected by the location of the fixed frame.

Example 2.1

Since line vectors are a subset of the set of screws, this example uses them to show some results discussed above. Let $\underline{\$}_1$ and $\underline{\$}_2$ be two line vectors, (see Figure 2.1) then

$$\underline{\$}_1 = \begin{bmatrix} \underline{S}_1 \\ \underline{r}_1 \times \underline{S}_1 \end{bmatrix}, \text{ and } \qquad \underline{\$}_2 = \begin{bmatrix} \underline{S}_2 \\ \underline{r}_2 \times \underline{S}_2 \end{bmatrix}$$

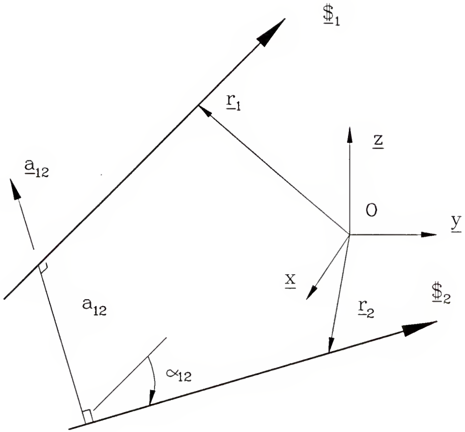

$\underline{\$}_1$
$\underline{r}_1$
$\underline{z}$
$\underline{a}_{12}$
O
$\underline{y}$
$\underline{x}$
$\underline{\$}_2$
a_{12}
$\underline{r}_2$
α_{12}

Figure 2.1

Addition of Screws

$$\underline{\$} = \underline{\$}_1 + \underline{\$}_2 = \begin{bmatrix} \underline{S}_1+\underline{S}_2 \\ \underline{r}_1\times\underline{S}_1+\underline{r}_2\times\underline{S}_2 \end{bmatrix}$$

Note that the screw $\underline{\$}$ is not necessarily a line vector ($h\neq0$), but it is a general screw.

Multiplication by a Scalar

$$\underline{\$} = \lambda\underline{\$}_1 = \begin{bmatrix} \lambda\underline{S}_1 \\ \lambda\underline{r}_1\times\underline{S}_1 \end{bmatrix}$$

The screw $\underline{\$}$ is still a line vector but is not unitized in this case.

Inner Product of Screws

$$\lambda = \underline{\$}_1^T\underline{\$}_2 = \underline{S}_1^T\underline{S}_2 + \underline{S}_{o1}^T\,\underline{S}_{o2}$$

$$= \underline{S}_1^T\,\underline{S}_2 + (\underline{r}_1\times\underline{S}_1)^T\,(\underline{r}_2\times\underline{S}_2) \tag{2.16}$$

When the scalar $\lambda=0$, the two screws are said to be orthogonal. A change of origin induces a change of the representation of the two line vectors and, specifically, the vectors $\underline{S}_{o1}$ and $\underline{S}_{o2}$.

Origin in O	Origin in A where $\overline{OA} = \underline{a}$
$\underline{S}_{o1}= \underline{r}_1\times\underline{S}_1$, and $\underline{S}_{o2}= \underline{r}_2\times\underline{S}_2$ $\lambda= \underline{S}_1^T\,\underline{S}_2 + (\underline{r}_1\times\underline{S}_1)^T\,(\underline{r}_2\times\underline{S}_2)$	$\underline{S}_{o1}'=(\underline{r}_1-\underline{a})\times\underline{S}_1$, and $\underline{S}_{o2}'=(\underline{r}_2-\underline{a})\times\underline{S}_2$ $\lambda'=\underline{S}_1^T\,\underline{S}_2+((\underline{r}_1-\underline{a})\times\underline{S}_1)^T\,((\underline{r}_2-\underline{a})\times\underline{S}_2)$ $\lambda' = \lambda - (\underline{a}\times\underline{S}_1)^T(\underline{r}_2\times\underline{S}_2) -$ $(\underline{a}\times\underline{S}_2)^T(\underline{r}_1\times\underline{S}_1) + (\underline{a}\times\underline{S}_1)^T(\underline{a}\times\underline{S}_2)$

Notice that when the two line vectors are orthogonal, i.e. $\lambda=0$, $\lambda' \neq 0$. Thus, the property of orthogonality between screws is made origin dependent by this kind of inner product. These considerations rule out the use of this kind of inner product to analyze screws. Instead, the reciprocal product, introduced in the following section, is physically meaningful and it is invariant with the change of origin. In this dissertation, the reciprocal product is the only product used, which leads to some interesting and novel results in the analysis of multi-fingered hands.

2.1.5 Reciprocal Screws

The reciprocal product $\mathcal{P}_{12}$ of two screws $\underline{\$}_1$ and $\underline{\$}_2$, is defined as

$$\mathcal{P}_{12} = \underline{S}_1^T \, \underline{S}_{o2} + \underline{S}_2^T \, \underline{S}_{o1} \tag{2.17}$$

where

$$\underline{\$}_1 = \begin{bmatrix} \underline{S}_1 \\ \underline{S}_{o1} \end{bmatrix}, \text{ and } \qquad \underline{\$}_2 = \begin{bmatrix} \underline{S}_2 \\ \underline{S}_{o2} \end{bmatrix}$$

Two screws are said to be reciprocal if and only if their reciprocal product $\mathcal{P}_{12}$ is equal to zero.

It is more convenient to write $\mathcal{P}_{12}$ as

$$\mathcal{P}_{12} = \underline{\$}_1^T \Delta \, \underline{\$}_2 \tag{2.18}$$

where Δ is 6×6 matrix introduced by Lipkin [1985]

$$\Delta = \begin{bmatrix} 0 & I_3 \\ I_3 & 0 \end{bmatrix} \tag{2.19}$$

In the case of two line vectors (see Figure 2.1)

$$\underline{\$}_1 = \begin{bmatrix} \underline{S}_1 \\ \underline{r}_1 \times \underline{S}_1 \end{bmatrix}, \text{ and } \qquad \underline{\$}_2 = \begin{bmatrix} \underline{S}_2 \\ \underline{r}_2 \times \underline{S}_2 \end{bmatrix} \quad .$$

$\mathcal{P}_{12}$, called also the mutual moment of the two lines, can be expressed as follows

$$\begin{aligned} \mathcal{P}_{12} &= \underline{S}_1^T(\underline{r}_2 \times \underline{S}_2) + (\underline{r}_1 \times \underline{S}_1)^T \underline{S}_2 \\ &= (\underline{r}_1 - \underline{r}_2)^T (\underline{S}_1 \times \underline{S}_2) \\ &= -a_{12} \sin(\alpha_{12}) \end{aligned} \tag{2.20}$$

where $$\underline{a}_{12} = \frac{\underline{S}_1 \times \underline{S}_2}{\|\underline{S}_1 \times \underline{S}_2\|} \tag{2.21}$$

a_{12} = the perpendicular distance (see Figure 2.1) between $\underline{\$}_1$ and $\underline{\$}_2$, and

α_{12} = the angle between $\underline{S}_1$ and $\underline{S}_2$.

For the mutual moment of two lines to be zero, they have to be either parallel (α_{12}=0) or intersecting (a_{12}=0).

For the general case of two screws, the reciprocal product $\mathcal{P}_{12}$ from Equation (2.17) can be written

$$\mathcal{P}_{12} = \underline{S}_1^T\ (\underline{r}_2 \times \underline{S}_2 + h_2 \underline{S}_2) + \underline{S}_2^T\ (\underline{r}_1 \times \underline{S}_1 + h_1 \underline{S}_1) \tag{2.22}$$

$$= (\underline{r}_1 - \underline{r}_2)^T (\underline{S}_1 \times \underline{S}_2) + (h_1 + h_2) \underline{S}_1^T\ \underline{S}_2 \tag{2.23}$$

$$\mathcal{P}_{12} = -a_{12} \sin(\alpha_{12}) + (h_1 + h_2) \cos(\alpha_{12}) \tag{2.24}$$

Two screws are said to be reciprocal if and only if $\mathcal{P}_{12} = 0$.

2.2 Introduction to the Kinestatics of Rigid Bodies

2.2.1 Instantaneous Kinematics of a Rigid Body

A revolute joint has an axis about which the rotation is possible. The ISA may be represented by a line vector. This line vector spans the screw system associated with the revolute joint. A prismatic joint is characterized only by its direction, so its ISA can be represented by a free vector, which spans the associated screw system. A screw joint combines rotation and translation, so it can be represented by a line vector and a free vector.

Example 2.2

Let b_1 be a body having a pure rotation ω_1 around $\underline{\$}_1$ and a pure translation τ_2 along $\underline{S}_2$, with respect to body b_2 (see Figure 2.2). The resulting motion can conveniently be represented by a screw $\omega\underline{\$}$ such that

$$\omega\underline{\$} = \omega_1 \begin{bmatrix} \underline{S}_1 \\ \underline{S}_{o1} \end{bmatrix} + \tau_2 \begin{bmatrix} \underline{0} \\ \underline{S}_2 \end{bmatrix} \tag{2.25}$$

$$\omega\underline{S} = \omega_1\underline{S}_1 \tag{2.26}$$

$$\omega\underline{S}_o = \omega_1\underline{r}_1\times\underline{S}_1 + \tau_2\underline{S}_2 \tag{2.27}$$

since

$$\|\underline{S}\| = \|\underline{S}_1\| = 1 \tag{2.28}$$

we get

$$\omega = \omega_1 \tag{2.29}$$

$$\underline{S} = \underline{S}_1 \tag{2.30}$$

$$\underline{S}_o = \underline{r}_1\times\underline{S}_1 + \frac{\tau_2}{\omega_1}\underline{S}_2 \tag{2.31}$$

$$h = \frac{\underline{S}_1{}^T\,\underline{S}_o}{\underline{S}^T\,\underline{S}} = \frac{\tau_2}{\omega_1}\underline{S}_1{}^T\,\underline{S}_2 \tag{2.32}$$

Ball called this motion a twist of amplitude ω on a screw $\underline{\$}$.

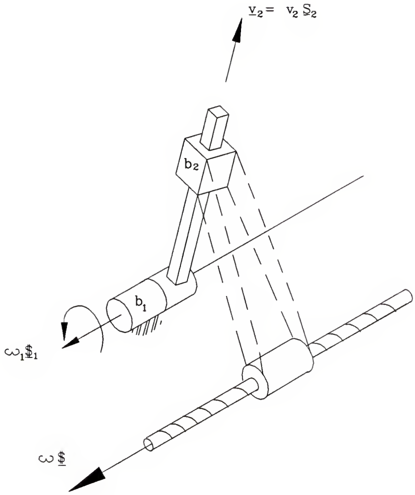

$\underline{v}_2 = v_2 \underline{S}_2$
b_2
b_1
$\omega_1 \underline{\$}_1$
$\omega \underline{\$}$

Figure 2.2

2.2.2 Statics of a Rigid Body

Analogous to instantaneous motions, unit line vectors can be used to express the action of a force an a body. A line vector with magnitude f and direction given by $\underline{\$}$ acting in the line passing through P ($\underline{r}_p = \overline{OP}$) is given by

$$f\underline{\$} = \begin{bmatrix} f\underline{S} \\ f\underline{r}_p \times \underline{S} \end{bmatrix} \tag{2.33}$$

A pure couple has no line of action, and may therefore be represented as a free vector. A general system of forces and couples acting upon a rigid body can be reduced to a single force

$$f_1\underline{\$}_1 = f_1 \begin{bmatrix} \underline{S}_1 \\ \underline{r}_1 \times \underline{S}_1 \end{bmatrix} \tag{2.34}$$

with a pure couple

$$C_2\underline{\$}_2 = C_2 \begin{bmatrix} \underline{0} \\ \underline{S}_2 \end{bmatrix} \tag{2.35}$$

This force-couple combination is represented by $f\underline{\$}$ such that

$$f = f_1 \tag{2.36}$$

$$\underline{S} = \underline{S}_1 \tag{2.37}$$

$$\underline{S}_o = \underline{r}_1 \times \underline{S}_1 + \frac{C_2}{f_1}\underline{S}_2 \tag{2.38}$$

$$h = \frac{C_2}{f_1}\underline{S}_1{}^T\,\underline{S}_2 \tag{2.39}$$

Ball called this resulting quantity a wrench of intensity f on a screw $\underline{\$}$.

2.2.3 Serially Connected Manipulators

This analysis is best described by an example.

Example 2.3

Given three bodies b_0, b_1, and b_2. b_1 has a relative motion with respect to b_0 represented by $\omega_1\underline{\$}_1$ and body b_2 moves with respect to b_1 along a screw $\omega_2\underline{\$}_2$. The instantaneous motion of b_2 with respect to b_0 is given by the screw $\omega\underline{\$}$ (Duffy [1980])

$$\omega\underline{\$} = \omega_1\underline{\$}_1 + \omega_2\underline{\$}_2. \tag{2.40}$$

This result can easily be generalized to a serial manipulator with n links

$$\omega\underline{\$} = \sum_{i=1}^{n}\omega_i\underline{\$}_i \tag{2.41}$$

where $\omega\underline{\$}$ represents the motion of the end effector with respect to the ground. On the other hand, the screw $\omega_i\underline{\$}_i$ represents the relative motion of the i^{th} link.

Serial manipulators are basically a succession of links connected together by joints (usually revolutes or prismatics). The first link is usually referred to as the base and is always fixed to a reference frame. The last link is called the end-effector of the robot.

Example 2.4

Figure 2.3 illustrates a typical industrial serial manipulator: the PUMA robot.

Given:

• The location of the end-effector $\underline{R}_p=[33,8,4]^T$ (in), pointing to a point P, and its orientation which can be represented by $\underline{a}_{67}=[0,1,1]^T$ and $\underline{S}_6=[0,1,-1]^T$,

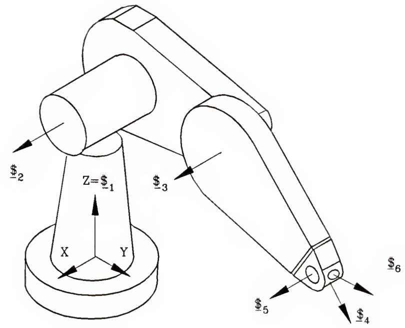

$\underline{\$}_2$
$Z=\underline{\$}_1$
$\underline{\$}_3$
X
Y
$\underline{\$}_6$
$\underline{\$}_5$
$\underline{\$}_4$

Figure 2.3

• The absolute velocity $\underline{V}_P=[10,-20,30]^T$(in/s) of a point P located on the end-effector and the angular velocity $\underline{\omega}=[100,-150,50]^T$ (deg/s) of the end-effector.

Find: The joint angular velocities ω_i's required to generate $(\underline{\omega},\underline{V}_P)$.

The motion of the end effector can be represented by a screw $\omega\underline{\$}$ where

$$\omega\underline{\$} = \begin{bmatrix} \underline{\omega} \\ \underline{R}_P \times \underline{\omega} \end{bmatrix} + \begin{bmatrix} \underline{0} \\ \underline{V}_P \end{bmatrix} \tag{2.42}$$

$$\omega\underline{\$} = \begin{bmatrix} 1.75 \\ -2.62 \\ .87 \\ 27.45 \\ -41.82 \\ -70.36 \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}1\\1\\1\end{matrix}}\right\} \text{rad/s} \\ \left.\vphantom{\begin{matrix}1\\1\\1\end{matrix}}\right\} \text{in/s} \end{matrix} \tag{2.43}$$

Since the end-effector is serially connected to the base, the screw equation of its motion can be written as follows:

$$\omega\underline{\$} = \sum_{i=1}^{6} \omega_i \underline{\$}_i \tag{2.44}$$

Equation (2.44) can be written in matrix form

$$\omega\underline{\$} = J\underline{\Omega} \tag{2.45}$$

where

$$J = \begin{bmatrix} \underline{\$}_1 & \underline{\$}_2 & \underline{\$}_3 & \underline{\$}_4 & \underline{\$}_5 & \underline{\$}_6 \end{bmatrix} \tag{2.46}$$

$$J = \begin{bmatrix} 0 & .42 & .42 & .74 & 0 & 1 \\ 0 & -.91 & -.91 & .34 & -.87 & 0 \\ 1 & 0 & 0 & -.59 & .5 & 0 \\ 0 & 0 & 6.4 & -5.4 & 3.2 & 0 \\ 0 & 0 & 2.9 & 14.2 & -13.4 & -2.3 \\ 0 & 0 & -14.8 & 1.4 & -23.4 & -10.4 \end{bmatrix}$$

J is called the Jacobian matrix of the robot.

$$\underline{\Omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \\ \omega_6 \end{bmatrix} \tag{2.47}$$

Solving for the joint angular velocities ω_i's from Equation (2.45), yields the following results

$$\underline{\Omega} = \begin{bmatrix} -45.75 \\ -18.12 \\ 112.46 \\ -149.20 \\ -16.17 \\ 170.58 \end{bmatrix} \text{deg/s}$$

2.2.4 Twists of Freedom and Wrenches of Constraint

The freedom of a rigid body can be represented by a screw system. A basis of this screw system represents all the possible motions of this rigid body. An unconstrained rigid body in space has six degrees of freedom, thus its motion is represented by a linear combination of a basis of the six-system. Adding n constraints on this rigid body leads to a reduction of the number of its degrees of freedom (DOF) to (6−n). A (6−n)-system can be found to represent any motion of the constrained rigid body. The remaining n degrees of freedom are now replaced by n constraints. Each constraint can be represented by a wrench on the constrained rigid body which is reciprocal to each and every element of the (6−n)-system of freedom.

In conclusion, the 6-sytem of screws can be divided in two subspaces one representing the twists of freedom, and the second representing the wrench of constraints.

Example 2.5

Figure 2.4 illustrates a constrained rigid body. The insertion of the peg in the hole can be described using the twists of freedom and the wrenches of constraint.

Twists of freedoms:

A rotation around the z-axis $\underline{T}_1 = [0, 0, 1; 0, 0, 0]^T$

A translation along the z-axis $\underline{T}_2 = [0, 0, 0; 0, 0, 1]^T$

Wreches of constraints:

A force along the x-axis $\underline{W}_1 = [1, 0, 0; 0, 0, 0]^T$

A force along the y-axis $\underline{W}_2 = [0, 1, 0; 0, 0, 0]^T$

A couple around x-axis $\underline{W}_2 = [0, 0, 0; 1, 0, 0]^T$

A couple around y-axis $\underline{W}_2 = [0, 0, 0; 0, 1, 0]^T$

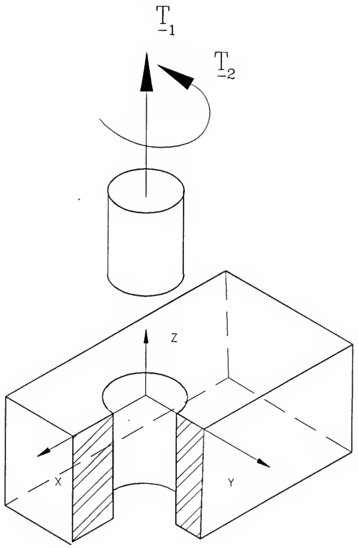


Figure 2.4

In this example, the peg has two DOF, and its motion is represented by a two-system spanned by $\underline{T}_1$ and $\underline{T}_2$. The 4-system spanned by $\{\underline{W}_1, \underline{W}_2, \underline{W}_3, \underline{W}_4\}$ repesents the constraints imposed by the environment. It also represents the wrenches that can be transmitted from the environment to the object without inducing any motion. Notice that each twist $\underline{T}$ is reciprocal to all the wrenches $\underline{W}_i$'s, which means that applying a wrench $\underline{W}_i$ to the peg does not produce any virtual power (Lipkin & Duffy [1986], and Griffis [1988]).

This hybrid decomposition of wrenches of constraints and twist of freedom was the subject of many discussions during the last decade. Mason, with his concept of "natural" and "artificial" constraints, used the concept of geometric segregation of Cartesian axes to generate the required hybrid decomposition (Mason [1978, 1979]). Also, he defined the center of compliance as the origin of cartesian axes. Several authors tried to apply this theory to control robot manipulators (Raibert & Craig [1981], Brady et al. [1982], Asada & Soltine [1986]). However, Paul [1987] and Ann & Hollerbach [1988] have shown through simulation as well as experimentation that this method of segregation yields unsatisfactory results.

In later work, Lipkin [1985], Lipkin & Duffy [1986] and Griffis [1988] showed that this decomposition is meaningless due to the use of the erroneous orthogonal product between screws (see Section 2.1.4). They also showed that the reciprocal product is the only product to be used between them. Furthermore, Griffis [1988] showed that other considerations must be addressed to have a complete and a meaningfull hybrid decomposition.

CHAPTER 3
KINESTATICS OF MULTI-FINGERED HANDS

3.1 Introduction

The main task of a hand is handling an object. By handling we mean grasping and manipulating an object using a set of fingers. The fingertips are the only contact between the fingers and the object. The motion of the grasped object is generated using the finger's joints. During this process the platform supporting the fingers, also called the palm, does not move.

In this section, a general hand of M fingers having N joints each, is investigated. A kinematic and a force analysis of the hand are developed. An example of a hand with M=3 and N=3 is chosen as a case study.

3.2 Type of Contact

Figure 3.1 shows a model of one finger contacting an object. Although several types of contacts between the object and the finger can be considered (see Salisbury [1982]), only point contacts with friction will be considered in this analysis. This choice stems from the feasablity and simplicity of this kind of contact. The kinematic analysis of a hand with this type of contact is investigated in the following section.

3.3 Instantaneous Kinematics of Multi-Fingered Hands

3.3.1 Basic Equation

Duffy [1980] modeled a spherical joint by three concurrent revolutes. Using this model, we can model a finger having a point contact with the object as a serial manipulator where the last three joints are three revolutes.

The twist $\underline{\$}$ $(=\omega\hat{\underline{\$}}$), representing the instantaneous motion of the grasped object of an M-fingered hand can be expressed as a linear combination of the N joint twists and the three contact twists in any of the fingers (see Figure 3.2) (see Maher & Duffy [1984])

$$\underline{\$} = \omega\hat{\underline{\$}} = \underbrace{\omega_1^{(i)}\underline{\$}_1^{(i)} + \ldots + \omega_N^{(i)}\underline{\$}_N^{(i)}}_{\text{Joint twists}} + \underbrace{\omega_{N+1}^{(i)}\underline{\$}_{N+1}^{(i)} + \ldots + \omega_{N+3}^{(i)}\underline{\$}_{N+3}^{(i)}}_{\text{Contact twists}} \qquad (3.1)$$

$i = 1 \ldots M,$

where $\omega_n^{(i)}$ denote the N known input scalar multipliers ($n=1,\ldots,N$).

The M simultaneous equations can be either solved for the screw motion $\underline{\$}$ or the joint angular velocities $\omega_n^{(i)}$. There are NM joint angular velocities, $\omega_n^{(i)}$, that are controllable (each one of these joints has a motor). For the case of three fingers and each finger has three joints, we have 3x3=9 joint angular velocities to solve for.

3.3.2 The Characteristic Matrix Equation

In each finger there are 3 unknown scalar multipliers, $\omega_n^{(i)}$, where $j= N+1, N+2, N+3$, and it is necessary to eliminate these unknowns. This is accomplished by forming a 3-system of screws which is reciprocal to the three-system of screws corresponding to the point contact. Taking the reciprocal product of both sides of Equation (3.1) with the appropriate reciprocal screw $\underline{\$}_{rj}^{(i)}$, for $i= 1,\ldots, M$, and $j= N+1, N+2, N+3$, gives 3M linear equations. This set of 3M equations can be written in matrix form as follows:

$$J^T\Delta\underline{\$} = K\underline{\Omega} \qquad (3.2)$$

where

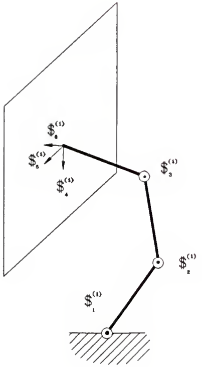


Figure 3.1

$$J = [\ \underline{\$}_{r1}^{(1)}\ \underline{\$}_{r2}^{(1)}\ \underline{\$}_{r3}^{(1)} \ldots \underline{\$}_{r1}^{(M)}\ \underline{\$}_{r2}^{(M)}\ \underline{\$}_{r3}^{(M)}\ ] \tag{3.3}$$

$$K = \begin{bmatrix} K^{(1)} & 0 & 0 & 0 \\ 0 & K^{(2)} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & K^{(M)} \end{bmatrix} \tag{3.4}$$

$$K^{(i)} = \begin{bmatrix} \underline{\$}_{r1}^{(i)T} \Delta \underline{\$}_1^{(i)} & \ldots & \underline{\$}_{r1}^{(i)T} \Delta \underline{\$}_N^{(i)} \\ \underline{\$}_{r3}^{(i)T} \Delta \underline{\$}_1^{(i)} & \ldots & \underline{\$}_{r3}^{(i)T} \Delta \underline{\$}_N^{(i)} \end{bmatrix} \tag{3.5}$$

$$\underline{\Omega} = \begin{bmatrix} \omega_1^{(1)} \\ \omega_2^{(1)} \\ \vdots \\ \omega_3^{(M)} \end{bmatrix} \tag{3.6}$$

$$\Delta = \begin{bmatrix} 0 & I_3 \\ I_3 & 0 \end{bmatrix} \tag{3.7}$$

and where

J is a 6×3M dual matrix

Δ is a 6×6	matrix given by equation (3.7)
K is a 3M×NM	matrix
$\underline{\Omega}$ is a NM×1	vector
$\underline{\$}$ is a 6×1	dual vector.

Equation (3.2) describes the instantaneous kinematics of a multi-fingered hands and gives the geometrical and analytical conditions for special configurations. This equation will be referred to as the Characteristic Matrix Equation of the hand.

When K is a square matrix it can be inverted to express $\underline{\Omega}$ directly in terms of the screw $\underline{\$}$ representing the motion of the object. However, this is not usually the case for grasping, and when K is a non-square matrix the well established Singular Value Decomposition (SVD) method is used to solve for (Steven [1980] and Appendix A). It is intersting to note that this procedure yields important information on singularities (special finger configuration).

It is important to recognize that the dual matrix J cannot be analyzed using classical linear algebra. This difficulty is overcome here by introducing a Modified Singular Value Decomposition (MSVD) for the matrix J (see Appendix B). This solution is valid when J is square or non-square matrix. Generally, the matrix J represents the contact wrenches. For example a single point contact with friction permits three rotations. There are three wrenches (forces) reciprocal to this instantaneous motion. Hence, for three fingers in point contacts the matrix J will be 6×9.

The MSVD, presented here, not only yields invariant solution for $\underline{\$}$ and important information on singularities (special grasping configurations) but also it yields the optimum motion of the grasped object.

The solution of the characteristic matrix equation for the screw $\underline{\$}$ yields the forward solution. On the other hand the reverse solution yields an expression of the joint angular velocities, $\underline{\Omega}$, in terms of the screw $\underline{\$}$.

3.3.3 Reverse Solution

In most viable grasping situations, the K matrix has more columns than rows (each finger has at least 3 joints; $N \geq 3$). Applying the SVD (see Appendix A) of the fully ranked matrix K^T in Equation (3.2) gives the following novel expression for the joint velocity vector $\underline{\Omega}$,

$$\underline{\Omega} = V_1 B_1^{-1} U_1^T J^T \Delta \underline{\$} + V_2 \underline{Y} \tag{3.8}$$

where $K^T = VBU^T$

and	V_1 is a $NM \times 3M$	Orthogonal matrix
		($V_1{}^T V_1 =$ Identity matrix)
	V_2 is a $NM \times (N-3)M$	Orthogonal matrix
		($V_2{}^T V_2 =$ Identity matrix)
	B_1 is a $3M \times 3M$	Diagonal matrix
	U_1 is a $3M \times 3M$	Orthogonal matrix
	$\underline{Y}$ is a $(N-3)M \times 1$	Arbitrary vector.

This is an explicit expression for the joint velocity vector $\underline{\Omega}$ in terms of the twist $\underline{\$}$ representing the instantaneous motion of the grasped object. When the object is stationary, $\underline{\$} = \underline{0}$ then $\underline{\Omega} = V_2 \underline{Y}$ is different from zero since the vector $\underline{Y}$ is arbitrary. This result is meaningful because it quantifies the redundancy in the hand viz. $\underline{Y}$ consists of $(N-3)M$ free parameters and therefore the hand possesses $(N-3)M$ degrees of redundancy. Redundancy is desirable because it permits more flexibility in the grasping

operation. There will be in general $\infty^{(N-3)M}$ hand configurations for a specified location of the object. Clearly, when $(N-3)M=0$ and there is no redundancy then there is only a single hand configuration ($\infty^0=1$). A hand with three joints in each finger is relatively easy to control since it has no flexibility. Hands with fingers with number of joints greater than three will have more flexibility but the control will be difficult. From here on only fingers with three joints will be considered ($N=3$) unless otherwise mentioned.

3.3.4 Forward Solution

Applying the MSVD of the matrix J in Equation (3.2) yields the expression of the twist $\underline{\$}$ of the grasped object directly in terms of the joint angular velocity $\underline{\Omega}$,

$$\underline{\$} = UB_1^{-1}V_1^TK\underline{\Omega} \tag{3.9}$$

together with

$$\underline{0} = V_2^T\underline{\Omega} \tag{3.10}$$

where (see Appendix B)

$$J = UV_1^T \tag{3.11}$$

U is a 6×6	Dual matrix
B_1 is a 6×6	Diagonal matrix
V_1 is a 3M×6	Orthogonal matrix
	($V_1^TV_1$ = Identity matrix)
V_2 is a 3M×(3M−6)	Orthogonal Matrix
	($V_2^TV_2$ = Identity matrix)

Equation (3.9) can be used to compute the actual motion of the grasped object and this can be compared with the specified motion in order to generate an error signal for a controller.

Equation (3.10) consists of (3M−6) linear equations which relate the scalar multipliers $\Omega_j^{(i)}$. There are only ever six free choices in the $\Omega_j^{(i)}$. For instance, for the three fingers M=3, there are nine $\Omega_j^{(i)}$'s and Equation (3.10) yields three linear equations, which results in only six free choices of the scalar multipliers $\Omega_j^{(i)}$.

3.4 Force Analysis

3.4.1 Grasp Analysis

A set of torques $\underline{\tau}$ must be generated by the joint motors to resist any external wrench $\underline{W}$ applied to the object.

$$\underline{\tau} = \begin{bmatrix} \tau_1^{(1)} \\ \tau_2^{(1)} \\ \vdots \\ \tau_3^{(3)} \end{bmatrix} \tag{3.12}$$

$$\underline{W} = \begin{bmatrix} \underline{F} \\ \underline{M} \end{bmatrix} \tag{3.13}$$

The rate of virtual work (or virtual power) can be expressed instantaneously as

$$\underline{\Omega}^T \underline{\tau} = \underline{\$}^T \Delta \underline{W} \tag{3.14}$$

Using the forward solution of Equation (3.2) yields

$$\underline{\tau} = K^T (V_1 B_1^{-1} U^T \Delta \underline{W} + V_2 \underline{\alpha}) \tag{3.15}$$

where $\underline{\alpha}$ is an arbitrary $(3M-6)\times 1$ vector.

$$\underline{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \tag{3.16}$$

The right side of Equation (3.15) has two terms, the first of which is a set of reaction (manipulation) torques due to the external wrench whilst the second is a set of torques due to the grasping wrenches. We will be refering to these two components as the manipulating torque and the grasping torque, respectively.

3.4.2 Contact Force Analysis

An in depth study of the wrenches transmitted by different types of contact has already been investigated by Salisbury [1982] and will not be repeated here.

The external wrench $\underline{W}$ on an object grasped by three fingers can be expressed in the following form:

$$\underline{W} = c_1^{(1)}\underline{W}_1^{(1)} + c_2^{(1)}\underline{W}_2^{(1)} + \ldots + c_3^{(3)}\underline{W}_3^{(3)} \tag{3.17}$$

where $\underline{W}_1^{(i)}$, $\underline{W}_2^{(i)}$, $\underline{W}_3^{(i)}$ are three linearly independent contact wrenches with intensities $c_1^{(i)}$, $c_2^{(i)}$, and $c_3^{(i)}$ respectively due to the contact of the i^{th} finger. For a point contact with friction these contact wrenches are simply three forces. $\underline{W}_1^{(i)}$ will denote the force normal to the plane of contact whilst $\underline{W}_2^{(i)}$ and $\underline{W}_3^{(i)}$ are in the plane of contact.

However, for a soft contact the contact wrenches consist of three forces together with a couple which is normal to the contact plane (see Salisbury [1982]).

At each contact the wrenches $\underline{W}_1^{(i)}$, $\underline{W}_2^{(i)}$, $\underline{W}_3^{(i)}$ are reciprocal to the twists $\underline{\$}_j^{(i)}$ allowed by the contact, and therefore (see also Equation (3.3))

$$J = [\ \underline{\$}_{r1}^{(1)} \quad \underline{\$}_{r2}^{(1)} \ \dots \ \underline{\$}_{r3}^{(3)}] \tag{3.18}$$

$$J = [\ \underline{W}_1^{(1)} \quad \underline{W}_2^{(1)} \ \dots \ \underline{W}_3^{(3)}] \tag{3.19}$$

Therefore Equation (3.17) can be expressed in the form

$$\underline{W} = J\underline{c} \tag{3.20}$$

$$\underline{c} = \begin{bmatrix} c_1^{(1)} \\ c_2^{(1)} \\ \vdots \\ c_3^{(3)} \end{bmatrix} \tag{3.21}$$

Solving Equation (3.21) for $\underline{c}$, using the MSVD of J (see Appendix B) yields the following explicit expression for $\underline{c}$

$$\underline{c} = V_1 B_1^{-1} U^T \Delta \underline{W} + V_2 \underline{\alpha} \tag{3.22}$$

Further comparing (3.22) and (3.15) yields

$$\underline{\tau} = K^T \underline{c} \tag{3.23}$$

By analogy with Equation (3.15), Equation (3.22) has two components of $\underline{c}$, of which the first is generated by the external wrench and the second is necessary for the grasp.

$$\underline{c} = \underline{c}_m + \underline{c}_g \tag{3.24}$$

where

$$\underline{c}_m = V_1 B_1^{-1} U^T \Delta \underline{W} \tag{3.25}$$

and

$$\underline{c}_g = V_2 \underline{\alpha} \tag{3.26}$$

3.4.3 Optimization

In order to secure the grasp the following conditions must be satisfied (Kerr [1984]):

- The intensities of the normal forces have to be positive. The normal forces are $\underline{W}_1^{(i)}$, i= 1 ... 3. The corresponding intensities thus satisfy

$$c_1^{(i)} \geq 0 \quad \text{for } i = 1 \ldots 3. \quad \boxed{1} \tag{3.27}$$

- For no slipping at the contact we need the following conditions

$$\left|c_2^{(i)}\right| \leq \mu \, c_1^{(i)} \qquad \boxed{2} \tag{3.28}$$

$$\left|c_3^{(i)}\right| \leq \mu\, c_1^{(i)} \qquad \boxed{3} \tag{3.29}$$

for i= 1 ... 3.

where μ is the coefficient of friction.

The vector $\underline{\alpha}$ must satisfy the above conditions for the grasp to be possible. In the case of three fingers with point contact the matrix V_2 is 9x3 and the above conditions can be written as

$$V_2\underline{\alpha} = \underline{c} - \underline{c}_m \tag{3.30}$$

Since conditions $\boxed{2}$ and $\boxed{3}$ contain the absolute value, they can be replaced by two conditions each

$$c_2^{(i)} \leq \mu\, c_1^{(i)} \qquad \boxed{2'} \tag{3.31}$$

$$-c_2^{(i)} \leq \mu\, c_1^{(i)} \qquad \boxed{2''} \tag{3.32}$$

$$c_3^{(i)} \leq \mu\, c_1^{(i)} \qquad \boxed{3'} \tag{3.33}$$

$$-c_3^{(i)} \leq \mu\, c_1^{(i)} \qquad \boxed{3''} \tag{3.34}$$

In total, there are 3 conditions given by $\boxed{1}$ and 6 by $\boxed{2}$ and 6 by $\boxed{3}$. This set of 15 linear inequalities can be solved using the simplex method of linear programming. The problem can be formulated in a matrix form as

$$\mathcal{A}\, \underline{\alpha} \leq \mathcal{B} \tag{3.35}$$

where

$$\begin{bmatrix} V_{21}^T \\ V_{22}^T-\mu V_{21}^T \\ -V_{22}^T-\mu V_{21}^T \\ V_{32}^T-\ \mu V_{31}^T \\ -V_{32}^T-\ \mu V_{31}^T \\ V_{24}^T \\ \vdots \\ V_{27}^T \\ \vdots \end{bmatrix} = \mathcal{A} \tag{3.36}$$

$\underline{V}_{2i}$ denotes the i^{th} column of the matrix V_2

$$\begin{bmatrix} -c_{m1}^1 \\ -c_{m2}^1-\mu c_{m1}^1 \\ c_{m2}^1-\mu c_{m1}^1 \\ -c_{m3}^1-\mu c_{m3}^1 \\ c_{m3}^1-\mu c_{m3}^1 \\ c_{m1}^2 \\ \vdots \\ c_{m1}^3 \\ \vdots \end{bmatrix} = \mathfrak{B} \tag{3.37}$$

The objective function to be minimized is the sum of the normal contact intensities

$$\min \mathfrak{C} = \min(\ c_1^{(1)} + c_1^{(2)} + c_1^{(3)}) \tag{3.38}$$

which can be expressed in terms of $\underline{\alpha}$ as

$$\min \mathcal{C} = \min[(\underline{V}_{21}^T + \underline{V}_{24}^T + \underline{V}_{27}^T) \underline{\alpha}] \tag{3.39}$$

This linear optimization is just an approximation of the real problem (see Kerr [1985]) since conditions $\boxed{2}$ and $\boxed{3}$ are a linearization of the following law

$$T^{(i)} \leq \mu N^{(i)} \tag{3.40}$$

where $T^{(i)}$ is the tangential force at the i^{th} contact

$$T^{(i)} = \sqrt{\left(c_2^{(i)}\right)^2 + \left(c_3^{(i)}\right)^2} \tag{3.41}$$

and $N^{(i)}$ is the normal force at the same contact

$$N^{(i)} = c_1^{(i)} \tag{3.42}$$

<u>Example</u> 3.1

Consider an object grasped by three fingers, and subject to its own weight, which can be represented by

$$\underline{W} = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.43}$$

and the coefficient of friction μ is equal to 0.333, the matrix J is given by

$$J = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The MSVD of the matrix J is given by

$$J = UV_1^T$$

where

$$U = \begin{bmatrix} 0 & -1.2 & 0 & 0 & -1.22 & 0 \\ 0 & 0 & 1.22 & 1.22 & 0 & 0 \\ -1.22 & 0 & 0 & 0 & 0 & -1.22 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ .41 & 0 & -.577 & .577 & 0 & .41 \\ 1.15 & 0 & .41 & .41 & 0 & -1.15 \end{bmatrix}$$

$$V_1 = \left[\begin{array}{cccccc} 0 & .41 & 0 & 0 & .41 & 0 \\ .29 & 0 & .41 & 0 & 0 & -.29 \\ -.41 & 0 & .58 & -.58 & 0 & -.41 \\ \hline -.14 & 0 & .41 & .41 & 0 & .14 \\ .43 & -.41 & 0 & 0 & -.41 & -.43 \\ -.41 & -.5 & -.29 & .29 & .5 & -.41 \\ \hline .14 & 0 & -.41 & -.41 & 0 & -.14 \\ .43 & .41 & 0 & 0 & .41 & .43 \\ -.41 & .5 & -.29 & .29 & -.5 & -.41 \end{array}\right]$$

$$V_2 = \left|\begin{array}{ccc} .23 & .04 & -.78 \\ -.63 & -.23 & -.19 \\ 0 & 0 & 0 \\ .09 & .78 & .06 \\ .43 & .13 & .29 \\ 0 & 0 & 0 \\ -.55 & .55 & -.13 \\ .2 & .09 & .49 \\ 0 & 0 & 0 \end{array}\right|$$

B= diag(-2.83, -2.45, -1.41, 1.41, 2.45, 2.83)

From Equation (3.25)

$$\underset{\sim}{c}_m = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 0 \\ 0 \\ 2 \end{bmatrix}$$

and the optimization process on Equation (3.35) yields

$$\underset{\sim}{\alpha} = \begin{bmatrix} 6 \\ 5.95 \\ 1 \end{bmatrix}$$

and using Equation (3.26), we get

$$\underset{\sim}{c}_g = \begin{bmatrix} .1 \\ -.03 \\ 0 \\ 6.03 \\ .07 \\ 0 \\ 6 \\ -.03 \\ 0 \end{bmatrix}$$

and

$$\underline{c} = \underline{c}_m + \underline{c}_g = \begin{bmatrix} .1 \\ -.03 \\ 0 \\ 6.03 \\ -.07 \\ 2 \\ 6 \\ .03 \\ 2 \end{bmatrix}$$

This is the set of minimum contact intensities necessary to securely grasp the object.

Notice that the value of $c_1^{(1)}$ is not zero due to an artificial constraint which sets the minimum value of the normal forces to 0.1. This condition assures a permanent contact between the fingertip and the object.

CHAPTER 4
SINGULARITIES AND SPECIAL CONFIGURATIONS

The term "Special Configuration" was introduced by Hunt [1978] and it is used to describe various unique geometrical phenomena which occur in the motion of planar and spacial mechanisms. An in depth discussion of the special configuration of serial manipulators can be found in Duffy [1981] and Maher [1983]. A brief discussion of this subject is presented in the next section.

4.1 Special Configurations of Serial Manipulators

Generally, for serial manipulators, these special configurations could be categorized into "Stationary Configurations" and "Uncertainty Configurations". In addition to these two types, an arrangement of the link lengths may form an immovable structure(see Figures 4.1, 4.2, and 4.3). Notice that the stationary configuration is the one more likely to happen during a path planning process also the immovable structure case is usually of no interest because it occurs only under rare circumstances.

4.1.1 Stationary Configurations

A stationary configuration is referred to sometimes as an "extreme-range position" (Duffy & Sugimoto [1981]). A stationary configuration of the planar four link mechanism are widely understood and one such configuration is illustrated by Figure 4.1.

This stationary configuration can be explained using the screw equation, given by Equation (2.40) which is written here for a four bar mechanism as follows

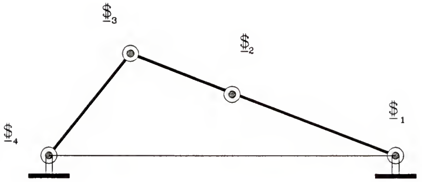


Figure 4.1

Figure 4.2

Figure 4.3

$$-\omega_4\underline{\$}_4 = \omega_1\underline{\$}_1+\omega_2\underline{\$}_2+\omega_3\underline{\$}_3 \tag{4.1}$$

The ISA (Instantaneous Screw Axis) of $\underline{\$}_1$, $\underline{\$}_2$ and $\underline{\$}_3$ all lie in the same plane in this case. The three screws thus belong to a two-system and

$$\underline{0} = \omega_1\underline{\$}_1+\omega_2\underline{\$}_2+\omega_3\underline{\$}_3 \tag{4.2}$$

Comparing (4.1) and (4.2),

$$\omega_4\underline{\$}_4 = \underline{0} \tag{4.3}$$

Therefore, $\omega_4=0$ and if $\omega_4\underline{\$}_4$ represents the instant motion of the end-effector (link a_{34}) of a 3R planar robot maniplator then the end-effector is stationary.

The stationary configuration of a serial spacial mechanism is very similar to its planar conterpart which is in this case a 6R serial manipulator and its equivalent 7R closed loop mechanism (Duffy [1980]). The screw equation in this case is given again by Equation (2.43) or (2.44) and is written here as

$$-\omega_7\underline{\$}_7 = \sum_{i=1}^{6}\omega_i\underline{\$}_i \tag{4.4}$$

When the rank of $J=[\underline{\$}_1\ \underline{\$}_2\ \cdots\ \underline{\$}_6]$ becomes less than six meaning that the six screws are linearly dependent, we can write

$$\underline{0} = \sum_{i=1}^{6}\omega_i\underline{\$}_i \tag{4.5}$$

and comparing (4.4) and (4.5)

$$\omega_7 \underline{\$}_7 = \underline{0} \tag{4.6}$$

$-\omega_7 \underline{\$}_7$ represents the motion of the end-effector of a 6R serial manipulator which is stationary in this configuration. This configuration is also an extreme-reach position for the robot and the end-effector is laying on the boundary of its workspace (see Sugimoto & Duffy [1981]).

In some cases two or more joints can become stationary at the same time. In general, when any n screws of a single loop mechanism with j joints become linearly dependent instantaneously, then the n screws form a configuration with mobility one. The remaining (j−n) linearly independent screws form a configuration which cannot move instantaneously. The mechanism is said to be in stationary configuration which can be determined by monitoring all the 6×6 determinants of the following 7×6 matrix

$$\begin{bmatrix} \underline{\$}_1^T \\ \underline{\$}_2^T \\ \underline{\$}_3^T \\ \underline{\$}_4^T \\ \underline{\$}_5^T \\ \underline{\$}_6^T \\ \underline{\$}_7^T \end{bmatrix} \tag{4.7}$$

A k^{th} joint is inactive when the 6×6 determinant which <u>does not</u> include the k^{th} row is zero.

4.1.2 Uncertainty Configurations

Here again we illustrate this case by an example of four bar planar mechanism. Consider that the axes of all screws representing the joint motions of the 4R mechanism lie in the same plane as illustrated by Figure 4.2, and that the input link is being driven at constant velocity ω_4 in counter-clockwise direction. The resulting motion of the coupler link a_{23} is instantaneously uncertain because, depending on the inertial loading on the system, it may rotate either clockwise or anti-clockwise. The mechanism is said to be in an uncertainty configuration.

In this case the four screws, representing the 4R belong to a two system because they all lie in one plane. Therefore, the screw equation, given by (4.1) and rewritten here as

$$-\omega_4\underline{\$}_4 = \omega_1\underline{\$}_1+\omega_2\underline{\$}_2+\omega_3\underline{\$}_3 \tag{4.8}$$

is equivalent to only two equations in the three unknowns ω_1, ω_2, and ω_3. To solve these equations an additional choice of one of the ω's is required. Thus, in this configuration the system becomes a two degree-of-freedom system.

For the case of seven screws, representing a 7R closed loop mechanism, the system is said to be in an uncertainty configuration if all the determinants of the 7×6 matrix, given by Equation (4.7), vanish simultaneously. It is not possible to solve the screw equation,

$$-\omega_7\underline{\$}_7 = \sum_{i=1}^{6}\omega_i\underline{\$}_i \tag{4.9}$$

for the six joint angular velocities, and the end-effector will hesitate or stop. Equation (4.9) can be expressed as a set of five non-homogeneous equations in five joint velocities, for instance ω_1, ω_2, $\cdots$, ω_5 and there is a free choice for the velocity ω_6.

Hunt [1978] classified uncertainty configurations according to the rank of the 7×6 matrix (Equation (4.7)), and a j-fold uncertainty configuration occurs when the rank of J is (6−j). Whilst the explanation of some uncertainty configurations is simple, others are more difficult to explain.

4.2 Special Configuration of Multi-Fingered Hands

The analysis presented in Chapter 3 will fail when the matrices J and/or K become singular. The matrix K is singular when one or more of the fingers is in special configuration. The matrix J is singular when the contact wrenches do not span a six space and the object being grasped is not fully constrained by the fingers. These are a total of four cases which must be considered:

4.2.1 Case 1: The matrix K is not fully ranked

Consider that the rank p of K is less than 9 (p<9). The reverse solution, using the SVD of K yields:

$$\underline{\Omega} = V_1B_1^{-1}U_1^TJ^T\Delta\underline{\$} + V_2\underline{Y} \tag{4.10}$$

together with

$$\underline{0} = U_2^TJ^T\Delta\underline{\$} \tag{4.11}$$

where $$K^T = VBU^T \tag{4.12}$$

U_1 is a $9\times p$ orthogonal matrix (4.13)

U_2 is a $9\times(9-p)$ orthogonal matrix (4.14)

These two equations are very important because they describe the kinematics of the fingers when singularities occur.

In Equation (4.11), JU_2 is a $6\times(9-p)$ dual matrix of the constrained wrenches and which is reciprocal to $\underline{\$}$. The rank of JU_2 is thus a measure of the dgrees of freedom lost by the object.

This case is equivalent to the uncertainty configuration for serial manipulators. Because the number of equations in the nine $\Omega_j^{(i)}$'s is less than nine, one or more of these parameters have to be chosen arbitrary.

Example 4.1

An example of this case is given by Figure 4.4 which illustrates a two fingered hand grasping an object. Each finger has three joints which results into six joint velocities $\Omega_j^{(i)}$ to be found.

The contact wrenches are

$$\underline{\$}_{r1}^{(1)} = \underline{W}_1^{(1)} = [\,-1, 0, 0; 0, 0, 0]^T$$
$$\underline{\$}_{r2}^{(1)} = \underline{W}_2^{(1)} = [\,0, 1, 0; 0, 0, 1]^T$$
$$\underline{\$}_{r3}^{(1)} = \underline{W}_3^{(1)} = [\,0, 0, 1; 0, -1, 0]^T$$
$$\underline{\$}_{r1}^{(2)} = \underline{W}_1^{(2)} = [\,1, 0, 0; 0, 0, 0]^T$$
$$\underline{\$}_{r2}^{(2)} = \underline{W}_2^{(2)} = [\,0, -1, 0; 0, 0, 1]^T$$
$$\underline{\$}_{r3}^{(2)} = \underline{W}_3^{(2)} = [\,0, 0, 1; 0, 1, 0]^T$$

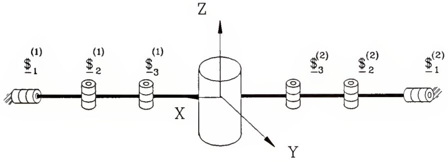

Z
$\underline{\$}_1^{(1)}$
$\underline{\$}_2^{(1)}$
$\underline{\$}_3^{(1)}$
$\underline{\$}_3^{(2)}$
$\underline{\$}_2^{(2)}$
$\underline{\$}_1^{(2)}$
X
Y

Figure 4.4

and the screws representing the joints are

$$\underline{\$}_1^{(1)} = [\ 1,\ 0,\ 0;\ 0,\ 0,\ 0]^T$$
$$\underline{\$}_2^{(1)} = [\ 0,\ 1,\ 0;\ 0,\ 0,\ 3]^T$$
$$\underline{\$}_3^{(1)} = [\ 0,\ 1,\ 0;\ 0,\ 0,\ 2]^T$$
$$\underline{\$}_1^{(2)} = [\ 1,\ 0,\ 0;\ 0,\ 0,\ 0]^T$$
$$\underline{\$}_2^{(2)} = [\ 0,\ 1,\ 0;\ 0,\ 0,\ -3]^T$$
$$\underline{\$}_3^{(2)} = [\ 0,\ 1,\ 0;\ 0,\ 0,\ -2]^T$$

The matrix K is given by Equation (3.4) and (3.5) and is rewritten here as

$$K = \begin{bmatrix} K^{(1)} & O \\ O & K^{(2)} \end{bmatrix} \tag{4.15}$$

where

$$K^{(1)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & 1 \end{bmatrix} \qquad K^{(2)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -2 & -1 \end{bmatrix} \tag{4.16}$$

The SVD of the matrix K yields (see Appendix A)

$$\underline{\Omega} = V_1 B_1^{-1} U_1^T J^T \Delta\underline{\$} + V_2 \underline{Y} \tag{4.17}$$

$$\underline{0} = U_2^T J^T \Delta\underline{\$} \tag{4.18}$$

$$JU_2 = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix}$$

The column of the matrix JU_2 represent the screws that cannot be generated by this grasp. The first and third columns represent the same screw which is a force along the x axis. The second and fourth columns are two forces passing through each contact point. In this case the singular grasp is unable to generate any rotation around the Y-axis.
Since the joints are pure rotors (as opposed to general screws) this condition implies that the fingers are in local or global extreme positions (see Sugimoto & Duffy [1981] and Waldron [1981]). By extreme positions it is meant that the fingers are fully extended (see Figure 4.4) or fully retracted, and in either case the tip of each finger lays on the boundary of its workspace.

Since this mechanism is a hybrid one, meaning that it is partially serial (each finger) and partially parallel mechanism (three fingers acting in parallel), some cases might occur which lead to some uncontrollable motions similar to the ones due to the "Singular Grasp".

4.2.2 Case 2: The matrix K is equal to zero

Equation (3.2) reduces to

$$J^T \Delta \underline{\$} = \underline{0} \tag{4.19}$$

When the rank of the matrix J is equal to six, the screw $\underline{\$}$, representing the motion of the object is reciprocal to all possible wrenches in space. Therefore, the motion of the grasped object is not possible and this result is independent of $\underline{\Omega}$.

Figure 4.5 shows an example when the matrix J has rank less than six, the grasped object will have (6−r) freedoms which are uncontrollable (a rotation around the axis of the two points of contact, in the case illustrated by the figure).

The case when K is equal to zero is similar to the case mentioned before of immovable structures for the serial manipulators, which is of little interest since it occurs only under very special circumstances (see Figure 4.4).

4.2.3 Case 3: The rank of the matrix J is less than 6

Consider that the rank r of J is less than 6 (r<6). Using the MSVD of J given in Appendix B and solving for the contact intensities yields

$$\underline{c} = V_1 B_1^{-1} U_1^T \Delta\underline{W} + V_2 \underline{\alpha} \tag{4.20}$$

together with

$$\underline{0} = U_2^T \Delta\underline{W} \tag{4.21}$$

where

U_1 is a 6xp ($p \leq r$) dual matrix of wrenches

U_2 is a 6×(6-p) dual matrix of wrenches and $\text{rank}(U_2)=6-r$

These pair of equations are important because they describe the status of grasping when the object is not fully constrained by the fingers.

In Equation (4.19) U_1 is a dual matrix of wrenches which can be generated by the "Singular Grasp".

In Equation (4.20) U_2 is a dual matrix of twists which cannot be restrained by the singular grasp and which are reciprocal to $\underline{W}$. The rank of U_2 is thus a measure of the relative freedom between the hand and the object.

Notice that the grasped object gains more freedom when J becomes singular, and the unrestrained motion is given by the matrix U_2, which is generated by the MSVD of J. On the other hand, it loses some of its freedom when the matrix K goes singular, and the additional constraints are given by the matrix JU_2, where U_2 is given by the SVD of the matrix K. This fact shows the duality between parallel and serial mechanisms.

Example 4.2

When a pair of fingers grasp an object with point contact the screws representing the contact wrenches are given in Example 4.1. The matrix J is

$$J = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

and the MSVD of J given by Appendix B yields

$$U_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

the columns of U_2 represent the twists that cannot be restrained by the grasp. In this example the rank of U_2 is equal to one and its column represents the twist around the line of the two contacting points (see Figure 4.4).

Due to the nature of the operator Δ, the rank of the matrix J (five in this case) is different from the rank of the matrix $J^T \Delta J$ (four in this case). In case of singularity the rank of the matrix J is equal to the degrees of freedom of the grasped object.

For point contact with friction there are three cases of singularities which depend on the number of fingers n. Equation (B-43), rewritten here as

$$0 = \mathrm{DET}\Big(n(R_1^2 + \cdots + R_n^2) - (R_1 + \cdots + R_n)^2 + \lambda^2 I\Big) \qquad (4.22)$$

yields always three pairs of $\pm\lambda_i$'s which are the modified singular values of the matrix J.

Let

$$\Psi = n(R_1^2 + \cdots + R_n^2) - (R_1 + \cdots + R_n)^2 \qquad (4.23)$$

a 3×3 matrix, three cases can be cosidered for different values of n:

• n=1, only one finger is contacting the object and the matrix $\Psi=0$. As we will show later the contact intensities required to counteract a properly placed external force are independant of the λ's. Therefore, the object can still be able to withstand external forces applied at the point contact. But due to the nature of the contact, a force pulling the object away from the fingertip will cause the contact to break. Thus, the point contact with friction can be modeled as a spherical joint as long as the conditions given by Equation (3.37) are satisfied. Clearly, an object contacted by only one finger cannot withstand any external wrench, except under very special cases. A force pushing the object straight against the fingertip (see Figure 4.5), could be resisted by only one finger. However, this contact is clearly unstable.

• n=2, this case is given by Example 4.1. The pair of eigenvalues $\pm\lambda_i$ corresponding to the axis joining the two contact points is zero. From Equation (3.25) we can conclude that a couple around the same axis yields an infinite contact intensities. This is the only singularity that could occur to the grasp.

• n$\geq$3, the matrix Ψ is always non singular provided that the contact points are not colinear. The object is fully constrained by the contacting fingertips.

Another way to classify these singularities is by examining the matrix U given by Equation (B-47)

$$U=p\begin{bmatrix} \underline{v}_1 & \underline{v}_1 & \underline{v}_2 & \underline{v}_2 & \underline{v}_3 & \underline{v}_3 \\ R\underline{v}_1+b_1\underline{v}_1 & R\underline{v}_1-b_1\underline{v}_1 & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \qquad (4.24)$$

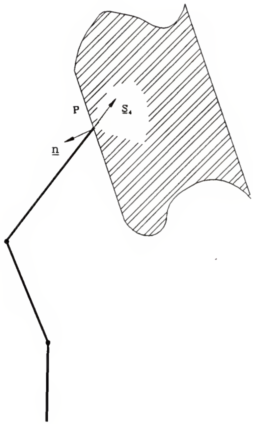


Figure 4.5

Three cases can be identified according to the values of the b_i's:

• All the b_i's are different from zero. In this case the matrix U is non singular and hence the grasp is non singular. Three or more fingers are required to achieve this kind of grasp.

• One of the b_i's, for instance b_1, is equal to zero. The matrix U is singular and can be split in two matrices U_1 and U_2 as follows:

$$U = [U_1 | \, U_2] \tag{4.25}$$

where

$$U_1 = \begin{bmatrix} \underline{v}_1 & \underline{v}_2 & \underline{v}_2 & \underline{v}_3 & \underline{v}_3 \\ R\underline{v}_1 & R\underline{v}_2 + b_2\underline{v}_2 & R\underline{v}_2 - b_2\underline{v}_2 & \cdots & \cdots \end{bmatrix} \tag{4.26}$$

$$U_2 = \begin{bmatrix} \underline{v}_1 \\ R\underline{v}_1 \end{bmatrix} \tag{4.27}$$

U_2 contains only one screw, which is a rotation around the axis joining the two contacting points (see Example 4.2).

• Two of the b_i's, for instance b_1 and b_2, are zero. The matrix U can be split in two matrices U_1 and U_2

$$U = [U_1 | \, U_2] \tag{4.28}$$

where

$$U_1 = \begin{bmatrix} \underline{v}_1 & \underline{v}_2 & \underline{v}_3 & \underline{v}_3 \\ R\underline{v}_1 & R\underline{v}_2 & R\underline{v}_3+b_3\underline{v}_3 & R\underline{v}_3-b_3\underline{v}_3 \end{bmatrix} \tag{4.29}$$

$$U_2 = \begin{bmatrix} \underline{v}_1 & \underline{v}_2 \\ R\underline{v}_1 & R\underline{v}_2 \end{bmatrix} \tag{4.30}$$

In this case two rotations are uncontrollable. The axes of these rotations are

$$\underline{\$}_{r1} = \begin{bmatrix} \underline{v}_1 \\ R\underline{v}_1 \end{bmatrix} \qquad \underline{\$}_{r2} = \begin{bmatrix} \underline{v}_2 \\ R\underline{v}_2 \end{bmatrix} \tag{4.31}$$

This kind of grasp is impossible to realize using only point contact with friction. This case is a good representation of a universal joint where two rotations around two perpendicular axes are allowed.

- All the three b_i's are zero. The matrix U is again decomposed in identical matrices U_1 and U_2

$$U = [U_1 | \, U_2] \tag{4.32}$$

$$U_1 = U_2 = \begin{bmatrix} \underline{v}_1 & \underline{v}_2 & \underline{v}_3 \\ R\underline{v}_1 & R\underline{v}_2 & R\underline{v}_3 \end{bmatrix} \tag{4.33}$$

In this case the three rotations are uncontrollable. This kind of grasp occurs when only one finger is contacting the object. The three screws in the matrix U_1 represent three

forces that can be transmitted through the contact. Also this matrix U can represent a spherical joint.

4.2.4 Case 4: J and K are Singular

Under these conditions the object has simultaneously some unattainable motions, defined by case 1, and some unrestrained ones, defined by case 3.

In the following section we will only consider the case where n=3, which allows a total control of the motion of the object in order to manipulate it properly.

CHAPTER 5
OPTIMAL GRASP

Several attempts were made to define an optimal grasp (Li & Sastry [1985], Kerr [1985]) and various criteria were proposed. Kerr used the singular values of the matrix J and decided that the best direction corresponds to the smallest one. But the singular value decomposition and the singular values are not invariant with a change of origin as noticed by Lipkin and Duffy [1985] and thus they are denied any physical meaning. Li & Sastry were aware of this problem and based their criteria on the task to be executed by the object. In this chapter the modified singular value decomposition, introduced in Appendix B, is used to generate the criteria needed for this optimization.

The problem of optimization can be stated as follows:

- Minimize $\underline{\Omega}$ (the joint angular velocities) or $\underline{c}$ (the contact intensities)
- Subject to a unit twist $\underline{U}$ of the object or a unit wrench $\underline{U}$ applied to the object.

In this formulation another problem arises which deals with the definition of a unit twist or wrench. In the next section a definition of a unit screw is presented based on the virtual power.

5.1 Unit Screws

In the following sections a novel analysis is proposed to optimize the motion of the grasped object. By this it is meant that it is required to minimize the instantaneous joint motions (or dually contact intensities) for a grasped object subjected to a unit twist (or dually a unit wrench).

Let the block in Figure 5.1 be moving along the direction represented by the unit vector $\underline{u}$. A force $\underline{F}$ is applied to the block which causes the motion along $\underline{u}$, and we have

$$\underline{F} = \lambda \underline{u} \tag{5.1}$$

$$\underline{v} = \upsilon \underline{u} \tag{5.2}$$

where $\underline{v}$ is the velocity of the block. The resultant virtual power is

$$P = \underline{F}^T \underline{v} = \lambda \upsilon \underline{u}^T \underline{u} \tag{5.3}$$

$$P = \lambda \upsilon \tag{5.4}$$

Therefore, for a unit force (λ=1) and a unit velocity (υ=1) the resultant virtual power is unity.

Analogously, for a body moving along a screw $\underline{\$}$ in space with a twist $\underline{T} = \omega \underline{\$}$ and subject to a wrench $\underline{W} = f\underline{\$}$ (see Figure 5.2), the virtual power P is (see Ball [188?]

$$P = \underline{W}^T \Delta \underline{T} = f\omega \underline{\$}^T \Delta \underline{\$} \tag{5.5}$$

For a unit force (f=1) and a unit twist (ω=1) the virtual power has to be unity, thus

$$\underline{\$}^T \Delta \underline{\$} = 1 \tag{5.6}$$

is the required condition for a screw to be a unit.

Before proceeding with this analysis it is necessary to define what is meant by a unit screw. This is not a trivial problem because associated with a twist (wrench) are the magnitudes of angular and translational velocities (force and couple).

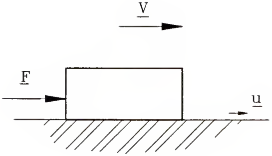


Figure 5.1

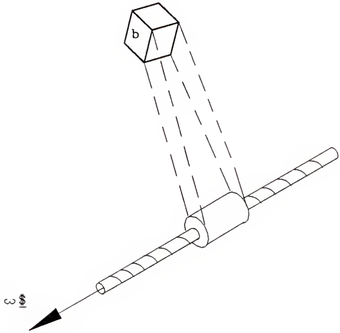


Figure 5.2

Kerr [1984] defined the magnitude by

$$\mathrm{mag}(\underline{\$}) = (\underline{\$}^T\underline{\$}) \tag{5.7}$$

This definition involves the orthogonal product of screws which has been shown to be non invariant and is therefore meaningless (Lipkin & Duffy [1984]).

Here we propose, using Equations (5.4) and (5.5),

$$\mathrm{mag}(\underline{\$}) = \sqrt{|\underline{\$}^T\Delta\underline{\$}|} \tag{5.8}$$

as the magnitude of a screw. We accept that $\underline{\$}^T\Delta\underline{\$}$ may be zero (i.e. when the screw is a line) or the pitch is infinite. In such instances we need to make respectively

$$\mathrm{mag}(\underline{\$}) = \|\underline{S}\| \tag{5.9}$$

and

$$\mathrm{mag}(\underline{\$}) = \|\underline{S}_o\| \tag{5.10}$$

respectively, these two agreeing with common earlier usage mentioned by Lipkin [1985].

As the pitch of a screw approaches zero or infinity, it is easy to see that there is a discontinuity in the assessment of the mag($\underline{\$}$), but in the event this does not affect the usefulness of this definition in the application described below.

It follows that we can now attach a meaning to a unit screw $\underline{U}$ along any screw of whatever pitch, namely

$$\underline{U} = \frac{\$}{\mathrm{mag}(\underline{\$})} \tag{5.11}$$

Let a body be constrained by a given unit screw $\underline{U}$ and having a twist $\underline{T} = \omega\underline{U}$ about it, where ω=mag($\underline{T}$). Let there be a wrench of the same pitch applied coaxially with $\underline{U}$; analogously $\underline{W}$=f$\underline{U}$ where f=mag($\underline{W}$). The virtual power of the wrench $\underline{W}$ acting on the rigid body moving along the twist $\underline{T}$ can be computed by (see Equation (5.5))

$$P = |\underline{W}^T \Delta \underline{T}| = f\theta |\underline{U}^T \Delta \underline{U}| = f\theta \ \mathrm{mag}(\underline{U}) = f\theta = \mathrm{mag}(\underline{W})\mathrm{mag}(\underline{T}) \tag{5.12}$$

In the degenerate case the screw $\underline{U}$ is reciprocal to itself and P=0. In this case, for the virtual power to be non-zero the wrench and the twist must be of different types; viz. when the wrench is on a line vector (force) then the twist has to be on a free vector (translation) and vice versa.

5.2 Optimum Grasp

Having assigned a meaning to a unit screw it is now possible to propose a formulation for the optimization of the motion of a grasped object. The problem can be stated as follows:

- Minimize ($\underline{\Omega}$)
- Subject to a unit twist $\underline{U}$ of the object

In other words it is requested to determine the unit twist representing the direction of motion of the grasped object which is produced by the minimum set of joint angular velocities. It is interesting to note that a unit wrench on the same screw requires the maximum set of joint torques to sustain it.

Dually, it is required to determine a unit wrench on the grasped object which requires the minimum set of joint torques. It is interesting to note that motion on this screw will generate the maximum set of joint velocities.

Consider that the motion of the grasped object is specified by a unit screw $\underline{U}(=[\underline{u},u_0]^T)$. In the case of three fingers grasping an object with point contacts, the matrix K is square and it can be inverted, and solving for $\underline{\Omega}$ Equation (3.2) yields,

$$\underline{\Omega} = J'^T \Delta \underline{\$} \tag{5.13}$$

where $J' = K^{-1}J$.

Since J' is a non-square matrix it is necessary to employ the modified SVD which has been derived in detail in Appendix B and which is expressed as $J'=UV_1^T$. Therefore,

$$\underline{\Omega} = V_1 U^T \Delta \underline{\$} \tag{5.14}$$

where (see Appendix B) V is a unit matrix,

$$V = [V_1 |\ V_2] = [\underline{V}_1 \cdots \underline{V}_6 |\ \underline{V}_7\ \ \underline{V}_8\ \underline{V}_9] \tag{5.15}$$

and U is a dual matrix,

$$U = [\underline{U}_1 \cdots \underline{U}_6] \tag{5.16}$$

The set of dual vectors $\{\underline{U}_i\}$ form a "co-reciprocal basis" for all possible motions and,

$$\underline{U}_i^T \Delta \underline{U}_j = b_{ij} \tag{5.17}$$

where

$$b_{ij} = b_i \qquad \text{when } i=j \qquad (5.18)$$

$$b_{ij} = 0 \qquad \text{when } i \neq j \qquad (5.19)$$

Let $\hat{\underline{U}}_i = \frac{\underline{U}_i}{\sqrt{|b_i|}}$ such that the screw $\hat{\underline{U}}_i$ is a unit screw

$$\left|\hat{\underline{U}}_i^T \Delta \hat{\underline{U}}_j\right| = \frac{\left|\underline{U}_i^T \Delta \underline{U}_j\right|}{|b_i|} = 1 \qquad (5.20)$$

Taking $\underline{\$}$ along one of the $\underline{U}_i$ in Equation (5.14) yields a simple expression for the joint rotations

$$\underline{\Omega} = \pm \underline{V}_i \sqrt{|b_i|} \text{ for } \underline{\$} = \underline{U}_i \qquad (5.21)$$

$$|\underline{\Omega}| = \|\underline{V}_i\| \sqrt{|b_i|} = \sqrt{|b_i|} = g_i \qquad (5.22)$$

and the minimium magnitude is

$$\min(\underline{\Omega}) = \min(g_i) = g \qquad (5.23)$$

In summary, a twist on $\hat{\underline{U}}_i$ corresponding to the minimum of g_i yields $\|\underline{\Omega}\|_{min}$, which constitutes the most sensitive twist of the object. On the other hand, a wrench on the same screw $\hat{\underline{U}}_i$ yields $\|\underline{\tau}\| = \frac{1}{\left|\sqrt{b_i}\right|} \left(\max(\|\underline{\tau}\|) = \frac{1}{\min(g_i)} = \frac{1}{g}\right)$. Therefore, the same screw $\underline{U}_i$ represents also the wrench which requires the maximum set of joint torques to sustain it.

This procedure can also be used to yield a quality measure to evaluate a grasp. Li & Sastry [1986] and Kerr [1984] proposed the use of the singular values of similar matrices to J' to evaluate the grasp. But the authors of Li & Sastry [1986] noticed that this criterion is not invariant with a change of origin and recommended that it should be avoided. This problem has been solved here by using the modified singular value decomposition (MSVD) and the modified singular values b_i as a criterion to choose between different grasps.

A choice of a grasp with a higher value of g yields a higher value of $\|\underline{\Omega}\|_{min}$, and a lower value of $\|\underline{\tau}\|_{max}$. Therefore, a compromize can be made in this case between sensitivity and strength of the grasp. For instance, if it is required to produce an accurate infinitesimal motion of the workpiece on some specified screw then high sensitivity (high g) is needed. This grasp will also yield low minimum joint torques and therefore the strength of the grasp is low.

On the other hand if it is required to move a heavy object for which contact forces are high (low g) then accurate manipulation may not be possible. This is clearly a problem of point contact with friction where normal pressures will be high due to the small contact areas. This problem is now discussed in some details.

5.3 Grasping with Point Contact with Friction

When an object is subjected to a high pressure, permanent deformations, or even destruction of the object, can take place during the grasping process. Further, when the grasped object bears a load, it may be distributed among all the fingertips. Hence, the efficiency of exerting a load on the object depends on the way it is grasped.

Consider the expression for the manipulating contact wrench intensities given by Equation (3.25) and also Appendix B

$$\underline{c}_m = V_1 B_1^{-1} U^T \Delta \underline{W} \tag{5.24}$$

where $\underline{W}$ is an external wrench acting upon the grasped object.

The MSVD of three point contacts with friction is given in detail in Appendix B. This analysis essentially reduces to determining the eigenvalues $\pm b_i$'s and the corresponding eigenvectors $\underline{v}_i$'s of the matrix

$$\Psi = 3\left(R_1^2 + R_2^2 + R_3^2\right) - \left(R_1 + R_2 + R_3\right)^2 \tag{5.25}$$

where

$$R_i = \begin{bmatrix} 0 & z_i & -y_i \\ -z_i & 0 & x_i \\ y_i & -x_i & 0 \end{bmatrix} = [\![\underline{r}_i \times]\!], \quad \text{for } i=1, 2, 3 \tag{5.26}$$

and $\underline{r}_i$ are vectors drawn from an origin O to the three contact points respectively.

Briefly U is a 6×6 matrix of the form

$$U = p \begin{bmatrix} \underline{v}_1 & \underline{v}_1 & \underline{v}_2 & \underline{v}_2 & \underline{v}_3 & \underline{v}_3 \\ R\underline{v}_1 + b_1\underline{v}_1 & R\underline{v}_1 - b_1\underline{v}_1 & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \tag{5.27}$$

where p and R are functions of the number n of fingers

$$p = \sqrt{\frac{n}{2}} \tag{5.28}$$

$$R = \frac{R_1+R_2+R_3}{n} \tag{5.29}$$

for the case of 3 fingers $p=\sqrt{\frac{3}{2}}$ and $R= \dfrac{R_1+R_2+R_3}{3}$.

Consider firstly that the wrench $\underline{W}$ is a unit force applied to a specified point C, defined by $\underline{OC}=\underline{r}= \dfrac{\underline{r}_1+\underline{r}_2+\underline{r}_3}{3}$ and that is parallel to one of the eigenvectors, for example $\underline{v}_1$. Therefore,

$$\underline{W} = \begin{bmatrix} \underline{v}_1 \\ R\underline{v}_1 \end{bmatrix} \tag{5.30}$$

where R is defined by Equation (5.29).

Substituting for $\underline{W}$ and U in (5.24) using (5.30) and (5.27) and simplifying the result yields, the following form for $\underline{c}_m$

$$\underline{c}_m = pV_1B_1^{-1} \begin{bmatrix} \frac{b_1}{3} \\ -\frac{b_1}{3} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5.31}$$

$$\underline{c}_m = p\frac{(\underline{V}_1+\underline{V}_2)}{3} \tag{5.32}$$

$$\|\underline{c}_m\| = \frac{p}{3}\sqrt{2} = \sqrt{\frac{1}{3}} \tag{5.33}$$

This is a most important result. A unit force applied to the special point C yields a constant value of the contact intensities, $\|\underline{c}_m\|$, which is independent of the direction of the applied force.

In general, for a hand with n fingers, the point C is defined by

$$\underline{r}_C = \frac{1}{n}\sum_{i=1}^{n}\underline{r}_i \tag{5.34}$$

and it is called the center of the grasp, and a force applied whose line of action passes through C is isotropic.

The center of the grasp is also the centroid of the contact points where all the points are weighted equally. By choosing the center of the grasp as close as possible to the point of application of the external force, we can minimize the contact intensities at the fingertips and have a constant value of the intensity of $\|\underline{c}_m\|$. Next, an analysis of the contact intensities, in the case of a pure couple as the external wrench, is presented.

Consider secondly that the wrench $\underline{W}$ is a unit couple applied to the object and it is along one of the eigenvectors, for example $\underline{v}_1$. Therefore,

$$\underline{W} = \begin{bmatrix} \underline{0} \\ \underline{v}_1 \end{bmatrix} \tag{5.35}$$

Substituting for $\underline{W}$ and U in (5.24) using (5.35) and (5.27) and simplifying the result

yields,

$$\underline{c}_m = pV_1B_1^{-1}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5.36}$$

$$= p\frac{(\underline{V}_1+\underline{V}_2)}{b_1} \tag{5.37}$$

$$\|\underline{c}_m\| = \frac{p}{b_1}\sqrt{2} = \frac{\sqrt{3}}{b_1} \tag{5.38}$$

Equation (5.38) shows that the minimum (maximum) $\|\underline{c}_m\|$ is produced by a pure couple along the eigenvector $\underline{v}_i$ corresponding to the maximum (minimum) eigenvalue b_i.

This result can be interpreted geometrically (Asada et al. [1987]) by an ellipsoid, which will be called here the Grasp Ellipsoid (see Figure 5.3). The principal axes are the eigenvectors $\underline{v}_1$, $\underline{v}_2$, and $\underline{v}_3$, and the length of each principal axis is equal to the reciprocal of the corresponding eigenvalue. The magnitude of the $\underline{c}_m$ will depend on the direction of $\underline{W}$ since the smallest (largest) eigenvalue corresponds to the major (minor) axis of the ellipsoid. The maximum (minimum) of $\|\underline{c}_m\|$ is due to a unit couple acting around the major (minor) axis of the ellipsoid. Therefore, a unit couple around the minor axis yields the minimum magnitude of the vector $\underline{c}_m$.

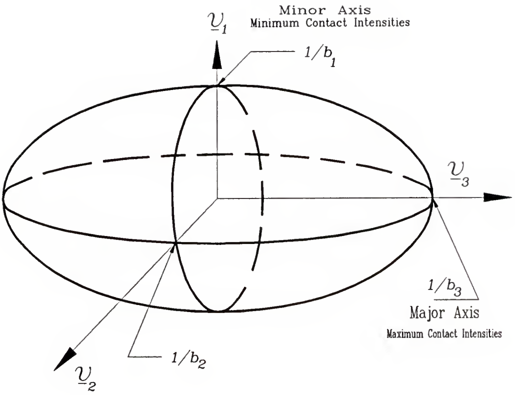


Figure 5.3: Grasp Ellipsoid

Example 5.1

Let the object shown in Figure 5.4 be grasped by three fingers. The matrix J in this case is given by

$$\underline{r}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{r}_2 = \begin{bmatrix} -.707 \\ .707 \\ 0 \end{bmatrix}$$

$$\underline{r}_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

The modified singular values of the matrix J are given by Equation (B-43)

$$\mathrm{DET}\Big(3\big(R_1^2 + R_2^2 + R_3^2\big) - \big(R_1 + R_2 + R_3\big)^2 + \lambda^2 I_3\Big) = 0$$

$$\mathrm{DET}\left(\begin{bmatrix} 4.4 & 1.4 & 0 \\ 1.4 & 4.4 & 0 \\ 0 & 0 & 8.8 \end{bmatrix} - \lambda^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = 0$$

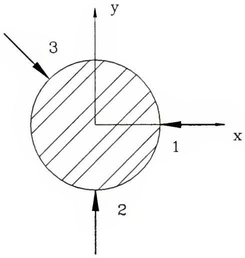


Figure 5.4

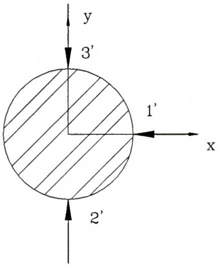


Figure 5.5

which yields

$$b_{1,2} = \pm\lambda_1 = \pm 1.73$$

$$b_{3,4} = \pm\lambda_2 = \pm 2.41$$

$$b_{5,6} = \pm\lambda_3 = \pm 2.97$$

and the corresponding eigenvectors

$$\underline{v}_1 = \begin{bmatrix} .707 \\ .707 \\ 0 \end{bmatrix}$$

$$\underline{v}_2 = \begin{bmatrix} .707 \\ -.707 \\ 0 \end{bmatrix}$$

$$\underline{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The grasp ellipsoid is shown in Figure 5.3. This analysis shows that a couple around the z-axis yields the minimum contact intensities at the fingertips. Therefore, this grasp is more efficient when the external wrench on the object can be reduced to a couple around

the z-axis. This also shows that this is the best way to grasp a nut mounted on a screw along the z-axis.

Let the same object grasped as shown in Figure 5.5, where the contact points are now located in $1'$, $2'$ and $3'$.

$$\underline{r}_{1'} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{r}_{2'} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\underline{r}_{3'} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

and the modified singular values are given by Equation (B-43)

$$\mathrm{DET}\left(3\left(R_1'^2 \ +R_2'^2 \ +R_3'^2 \right) \ - \left(R_1' + R_2' + R_3'\right)^2 + \lambda^2 I_3\right) = 0$$

$$\mathrm{DET}\left(\begin{bmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 8 \end{bmatrix} - \lambda^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = 0$$

which yields

$$b_{1,2} = \pm\lambda_1 = \pm 1.41$$

$$b_{3,4} = \pm\lambda_2 = \pm 2.45$$

$$b_{5,6} = \pm\lambda_3 = \pm 2.83$$

and the corresponding eigenvectors

$$\underline{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\underline{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Here again the z-axis is best axis to apply a couple to the grasped object.

Comparing the two grasps yields that the contact forces at the fingertips of the first grasp for which $\min(|b_i|) = 1.73$ are less than those for the second grasp for which

the $\min(|b_i|) = 1.41$. Further, since the grasp ellipsoid for the first grasp is smaller than the one for the second grasp, it yields smaller contact intensities overall.

Considering a different grasp, where the number of fingers is greater than three, say four, and not all the contacting points are on the same plane.

$$r_1'' = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$r_2'' = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$r_3'' = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$r_4'' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The modified singular values are given by Equation (B-43)

$$\mathrm{DET}\left(4\left(R_1''^2+R_2''^2+R_3''^2+R_4''^2\right) - \left(R_1''+R_2''+R_3''+R_4''\right)^2+\lambda^2 I_3\right)=0$$

$$\mathrm{DET}\left(\begin{bmatrix} 12 & 1 & 0 \\ -1 & 10 & 0 \\ 0 & 0 & 8 \end{bmatrix} - \lambda^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = 0$$

which yields

$$b_{1,2} = \pm\lambda_1 = \pm 2.45$$

$$b_{3,4} = \pm\lambda_2 = \pm 3.16$$

$$b_{5,6} = \pm\lambda_3 = \pm 3.46$$

and the corresponding eigenvectors

$$\underline{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\underline{v}_2 = \begin{bmatrix} 0.707 \\ 0 \\ 0.707 \end{bmatrix}$$

$$\underline{v}_3 = \begin{bmatrix} -0.707 \\ 0 \\ 0.707 \end{bmatrix}$$

5.4 General Criterion

A general criterion to chose between grasps can be defined as follows

$$\eta = \frac{\sum_{i=1}^{6} |b_i|}{2n} \tag{5.39}$$

where n is the number of fingers. Since the b_i's come in pairs, the definition of η can be simplified to yield

$$\eta = \frac{b_1 + b_3 + b_5}{n} \tag{5.40}$$

This criterion represents the efficiency of the grasp. A grasp with higher η is more efficient in resisting the external wrench applied to the grasped object. Moreover, this criterion allows the comparison of grasps with different numbers of fingers. The following example can best demonstrate the usefulness of this criterion.

Example 5.2

Using the same cases mentioned in the above example, we get

- for case 1 shown in Figure 5.4, where n=3

$$\left.\begin{aligned} b_1 &= 1.73 \\ b_3 &= 2.41 \\ b_5 &= 2.97 \end{aligned}\right\} \quad \eta_1 = 2.37$$

- for case 2, shown in Figure 5.5, where n=3

$$\left.\begin{aligned} b_1 &= 1.41 \\ b_3 &= 2.45 \\ b_5 &= 2.83 \end{aligned}\right\} \quad \eta_2 = 2.23$$

- Four finger, Figure 5.6

$$\left.\begin{array}{l} b_1 = 2.45 \\ b_3 = 3.16 \\ b_5 = 3.46 \end{array}\right\} \quad \eta_3 = 2.26$$

Clearly, case 1 yields the most efficient grasp since it has the highest η. It is interesting to note that, for case 3, increasing the number of fingers did not produce a more efficient grasp.

Other cases can be mentioned which will show that the number of fingers is not the best way to improve the quality of the grasp; but it is the way the fingers are placed around the object that makes the grasp more efficient.

- Four fingers, Figure 5.7,

$$\left.\begin{array}{l} b_1 = 2.83 \\ b_3 = 2.83 \\ b_5 = 4.00 \end{array}\right\} \quad \eta_4 = 2.41$$

- Five fingers, Figure 5.8,

$$\left.\begin{array}{l} b_1 = 3.74 \\ b_3 = 3.74 \\ b_5 = 4.47 \end{array}\right\} \quad \eta_5 = 2.39$$

The last two examples are "equivalent" in the sense that they have the same set of b_i's. However, these two grasps differ in the optimum directions, which can be shown by looking at the two corresponding grasp ellipsoids; the first one has the y-axis and the

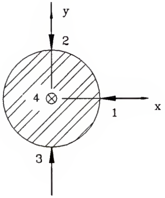


Figure 5.6

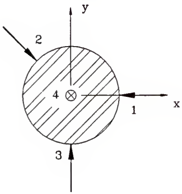


Figure 5.7

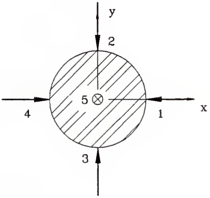


Figure 5.8

z-axis as major and minor axes respectively, while the second has the y-axis and the x-axis as major and minor axes respectively.

Figure 5.9 summarizes the above results. It is important to recognize that increasing the number of fingers does not necessarely improve the efficiency of the grasp.

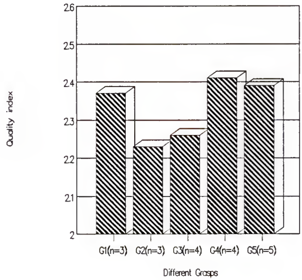


Figure 5.9

CHAPTER 6
PATH GENERATION AND COMPUTER GRAPHICS SIMULATION

6.1 Introduction

In the process of design, computer graphics simulation is a very important and rewarding step. This chapter is concerned with the design and animation of a mechanical hand model. Two main objectives are sought in a computer graphics simulation

- the diplay of different designs displayed via solid 3-D models,
- the display of each individual design executing a variety of tasks.

The comparison and evaluation of the graphics model enables us to choose the best design from a number of alternatives. This last concept was presented by Crane [1987] and was applied to robot manipulators.

In this chapter we will be more concerned with the second aspect of this problem, namely path planning. The calculation of a series of joint angles for each finger which will cause the grasped object to move along a user specified path, is presented here. These calculations will serve as input to the hand animation program, also presented in this chapter. In this manner, the user will be able to observe and evaluate the hand-object motion on the graphics screen. The model used for this demonstration is presented in section 6.2.2.

The first problem to be considered will be the location of the palm with respect to the object, which allows for the grasp and the manipulation of the object. The problem of path generation is then reduced to the selection of intermediate object positions and

orientations between the start and target positions. A reverse kinematic analysis is then performed for each finger for each of these locations.

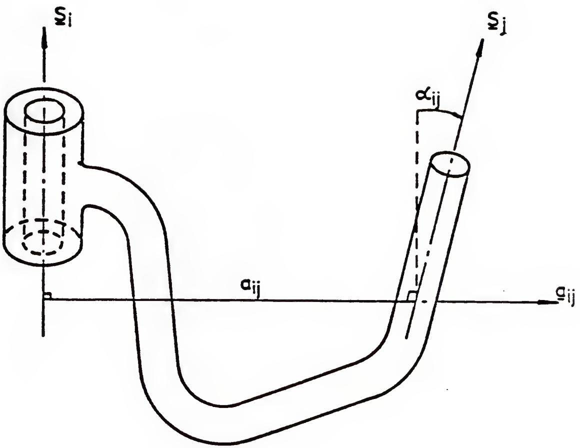
6.2 Kinematic and Force Analysis of the Hand

6.2.1 Notation

Each finger may be considered as a set of rigid bodies connected in a chain by joints. These bodies are called links. Many manipulators have sliding joints called prismatic joints. A single link of a typical robot (a robot is referred to as a finger in this chapter) has many attributes which a mechanical designer has to consider during its design. These include the type of material used, the strength and stiffness of the link, the external shape, the weight and inertia, etc. However, for the purposes of obtaining the kinematic equations of the finger, a link is considered only as a rigid body which defines the relationship between two neighboring joint axes of a finger. The notation used throughout this analysis is that developed by Duffy [1980]. One link, as the one shown in Figure 6.1, connects the two joint axes $\underline{S}_i$ and $\underline{S}_j$. The perpendicular distance berween them is well defined and is referred to as a_{ij} and lies along the mutual perpendicular $\underline{a}_{ij}$. The second parameter needed to define the relative location, is the twist angle α_{ij}. This angle is measured in a right handed sense about the vector $\underline{a}_{ij}$.

Neighboring links have a common joint axis between them. Figure 6.2 shows two links connected by a revolute joint. One parameter of interconnection has to do with the distance along this common axis from one link to the next and is referred to as S_{ii}. The second parameter describes the amount of rotation about this common axis between one link and its neighbor. This parameter is called the joint angle, and is referred to as θ_i.

Of the parameters joint angles (θ_i), twist angles (α_{ij}), offsets (S_{ii}), and link lengths (a_{ij}), only (θ_i) is a variable quantity while the remaining three are known constant values.



$\underset{\sim}{S}_i$
$\underset{\sim}{S}_j$
α_{ij}
a_{ij}
$\underset{\sim}{a}_{ij}$

Figure 6.1

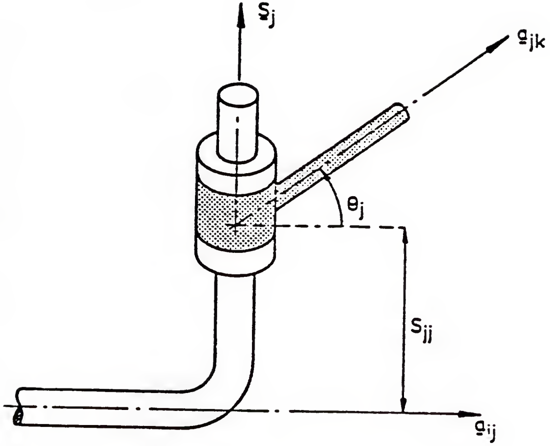


Figure 6.2

Using Figure 6.1 and Figure 6.2 it is possible to derive the following equations for the sine and cosine of α_{ij} and θ_i

$$\cos(\alpha_{ij}) = c_{ij} = \underline{S}_i^T \underline{S}_j \tag{6.1}$$

$$\sin(\alpha_{ij}) = s_{ij} = |\underline{S}_i \; \underline{S}_j \; \underline{a}_{ij}| \tag{6.2}$$

$$\cos(\theta_j) = c_j = \underline{a}_{ij}^T \underline{a}_{jk} \tag{6.3}$$

$$\sin(\theta_j) = s_j = |\underline{a}_{ij} \; \underline{a}_{jk} \; \underline{S}_j| \tag{6.4}$$

6-2-2 Dimension of the Hand Model

A palm and three identical fingers are the main components of the hand. The palm serves as a base for the three fingers. Shown in Figure 6.3[1] is a skeletal drawing of one of these fingers. Each finger is essentially a 3-R robot where the first joint is vertical, the second and third are parallel and at right angles to the first one. The skeletal model of the fingers is very similar to that for the T3-776, when the wrist is excluded [Crane 1987].

The following table contains the values of the constants:

$S_{22} = 0$	$a_{12} = 0$	$\alpha_{12} = 90^\circ$
$S_{33} = 0$	$a_{23} = 10.0$	$\alpha_{23} = 0^\circ$
$S_{44} = 10.0$	$a_{34} = 0$	$\alpha_{34} = 90^\circ$

The vector $\underline{a}_{ij}$ is defined as the normal to the plane defined by the two vectors $\underline{S}_i$ and $\underline{S}_j$ and is given by

1 For the sake of simplicity, all the superscripts (i) (i=1, 2, and 3) are omitted from the figures.

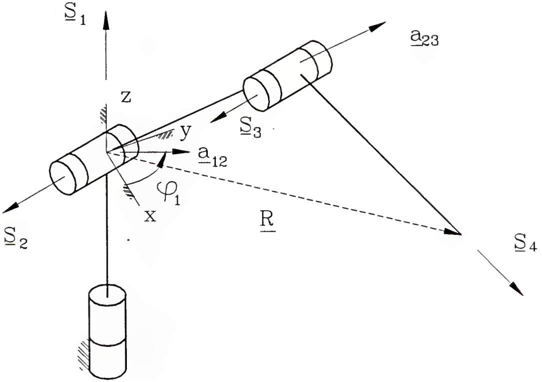


Figure 6.3

$$\underline{a}_{ij} = \frac{\underline{S}_i \times \underline{S}_j}{|\underline{S}_i \times \underline{S}_j|} \tag{6.5}$$

In the case where $\underline{S}_i$ is parallel to $\underline{S}_j$, $\underline{a}_{ij}$ is along the common perpendicular and pointing from $\underline{S}_i$ to $\underline{S}_j$.

6-2-3 Reverse Analysis

The reverse analysis of the hand can be reduced to a reverse analysis for each finger. The location of the finger contact points, relative to the grasped object, are assumed to be known. Following this, it is necessary to determine the positions of each finger tip in the fixed coordinate system. This is accomplished from the knowledge of the position and orientation of the grasped object, measured relative to the fixed reference. Finally, from the location of each fingertip $R^{(i)}$ (see Figure 6.3) the set of joint angles for each finger, $\phi_1^{(i)}$, $\theta_2^{(i)}$, and $\theta_3^{(i)}$ are computed. Given all these data, one can perform the reverse analysis of each finger following these steps:

- The vector $\underline{R}_f^{(i)}$, where f refers to the fixed coordinate system, is given by (see Figure 6.4)

$$\underline{R}_f^{(i)} = \underline{R}_f^{O} + A_{Of}\underline{r}_i^{(i)} \tag{6.6}$$

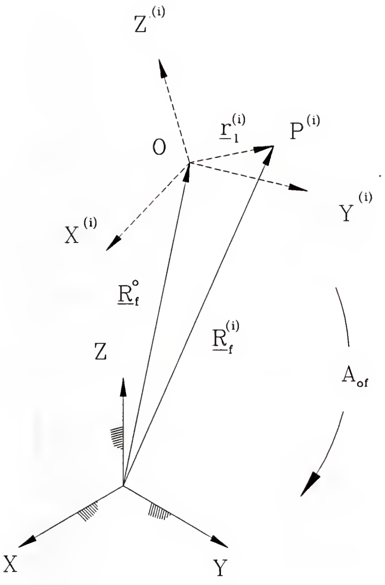
where

$\underline{R}_f^{O}$ is the vector pointing to origin of the object in the fixed coordinate system

A_{Of} is a rotation matrix from the local coordinate system of the object to the fixed one.

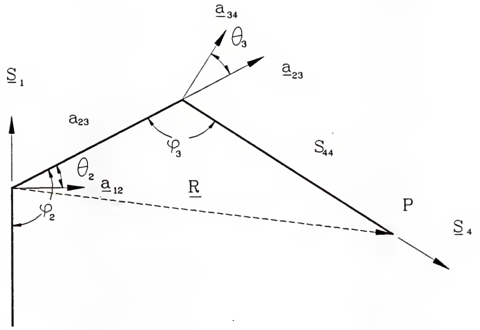
$\underline{r}_i^{(i)}$ represents the local position of the i^{th} contact point.

- Figure 6.5 illustrates that the vectors $\underline{R}_f^{(i)}$, $\underline{a}_{12}^{(i)}$, $\underline{a}_{23}^{(i)}$, $\underline{S}_4^{(i)}$, and $\underline{S}_1^{(i)}$ all lie in the same plane. Because of this, simple planar relationships can be utilized to determine the three relative displacement angles $\phi_1^{(i)}$, $\theta_2^{(i)}$, and $\theta_3^{(i)}$ for each finger.



$Z^{(i)}$
$\underline{r}_1^{(i)}$
$P^{(i)}$
O
$Y^{(i)}$
$X^{(i)}$
$\underline{R}_f^{\circ}$
$\underline{R}_f^{(i)}$
Z
A_{of}
X
Y

Figure 6.4



$\underline{a}_{34}$
θ_3
$\underline{a}_{23}$
$\underline{S}_1$
a_{23}
φ_3
S_{44}
θ_2
$\underline{a}_{12}$
$\underline{R}$
P
$\underline{S}_4$
φ_2

Figure 6.5

The angle $\phi_1^{(i)}$ is defined as the angle between the fixed x−axis and the vector $\underline{a}_{12}^{(i)}$ measured as a positive twist about the vector $\underline{S}_1^{(i)}$. The following equations are used to compute the sine and cosine of $\phi_1^{(i)}$,

$$\cos(\phi_1^{(i)}) = \underline{i}^T \underline{a}_{12} \tag{6.7}$$

$$\sin(\phi_1^{(i)}) = \left| \underline{i}\ \underline{a}_{12}^{(i)}\ \underline{S}_1^{(i)} \right| \tag{6.8}$$

where the vector $\underline{a}_{12}^{(i)}$ is given by

$$\underline{a}_{12}^{(i)} = \pm \frac{(\underline{R}_f^{(i)T}\underline{i})\underline{i} + (\underline{R}_f^{(i)T}\underline{i})\underline{i}}{\sqrt{(\underline{R}_f^{(i)T}\underline{i})^2 + (\underline{R}_f^{(i)T}\underline{i})^2}} \tag{6.9}$$

With the above convention for $\underline{a}_{12}^{(i)}$ two values of $\phi_1^{(i)}$ are given by Equations (6.7) and (6.8). However, only the value of $\phi_1^{(i)}$ in the interval [0, π] is considered in this analysis.

Two extra variables, $\phi_2^{(i)}$, and $\phi_3^{(i)}$, are introduced (see Figure 6.5) in order to facilitate the analysis. These angles are related to the angles $\theta_2^{(i)}$ and $\theta_3^{(i)}$ by the following equations:

$$\theta_2^{(i)} = \phi_2^{(i)} - 90^o \tag{6.10}$$

$$\theta_3^{(i)} = \phi_3^{(i)} - 90^o \tag{6.11}$$

Applying the cosine law to the planar triangle (see Figure 6.5) yields

$$\cos(\phi_3^{(i)}) = \frac{(a_{23}^{(i)2} + S_{44}^{(i)2} - \|\underline{R}^{(i)}\|^2)}{2a_{23}^{(i)}S_{44}^{(i)}} \tag{6.12}$$

and hence two values for $\phi_3^{(i)}$, viz. $\pm\phi_3^{(i)}$, which define two configurations commonly referred to as "elbow up" and "elbow down". Figure 6.6 shows a single finger contacting an object in its two configurations. Because of the finite size of the grasped object either of the configurations may not be possible because of the interference with the object, and there are three possibilities. Consider now that $\underline{n}^{(i)}$ is the outer normal to the contacting plane at the point of contact $P^{(i)}$,

case 1: $\underline{S}_4^{(i)T}\underline{n}^{(i)} < 0$ for both values of $\phi_3^{(i)}$ (see Figure 6.6)

The value with the higher $(\underline{S}^{(i)T}\underline{n}^{(i)})$ yields a "better" grasp. By better grasp it is meant that the potential range of motion of the grasped object is bigger.

case 2: $\underline{S}_4^{(i)T}\underline{n}^{(i)} < 0$ for one of the values for $\phi_3^{(i)}$ (see Figure 6.7)

Therefore, there is only one grasp configuration.

case 3: $\underline{S}_4^{(i)T}\underline{n}^{(i)} > 0$ for both values of $\phi_3^{(i)}$ (see Figure 6.8)

No real grasp configuration is possible is this case.

The final step in this analysis is the computation of the joint angle $\theta_2^{(i)}$. Projection is used to derive the necessary equations. It is shown in Figure 6.5 that the vector $\underline{R}_f^{(i)}$ can be written as

$$\underline{R}_f^{(i)} = a_{23}^{(i)}\,\underline{a}_{23}^{(i)} + S_{44}^{(i)}\,\underline{S}_4^{(i)} \tag{6.13}$$

Forming the scalar product of (6.13) with $\underline{a}_{12}^{(i)}$ and then with $\underline{S}_1^{(i)}$ and manipulating the equations, one gets the following two equations in $c_2^{(i)}$ and $s_2^{(i)}$,

$$\underline{R}_f^{(i)T}\,\underline{a}_{12}^{(i)} = c_2^{(i)}\left(a_{23}^{(i)} - S_{44}^{(i)}\cos(\phi_3^{(i)})\right) + s_2^{(i)}\,S_{44}^{(i)}\sin(\phi_3^{(i)}) \tag{6.14}$$

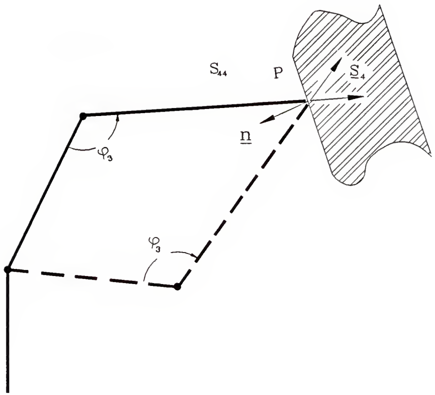


Figure 6.6

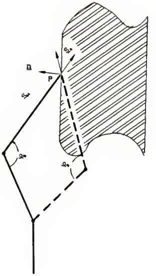


Figure 6.7

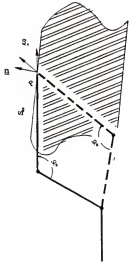


Figure 6.8

$$\underline{R}_f^{(i)T}\, \underline{S}_1^{(i)} = -c_2^{(i)}\, S_{44}^{(i)} \sin(\phi_3^{(i)}) + s_2^{(i)}\Big(a_{23}^{(i)} - S_{44}^{(i)}\cos(\phi_3^{(i)})\Big) \tag{6.15}$$

and solving for $c_2^{(i)}$ and $s_2^{(i)}$ yields

$$c_2^{(i)} = \frac{\Big(a_{23}^{(i)} - S_{44}^{(i)}\cos(\phi_3^{(i)})\Big)(\underline{R}_f^{(i)T}\, \underline{a}_{12}^{(i)}) - S_{44}^{(i)} \sin(\phi_3^{(i)})(\underline{R}_f^{(i)T}\, \underline{S}_1^{(i)})}{\|\underline{R}_f^{(i)}\|^2} \tag{6.16}$$

$$s_2^{(i)} = \frac{\Big(a_{23}^{(i)} - S_{44}^{(i)}\cos(\phi_3^{(i)})\Big)(\underline{R}_f^{(i)T}\, \underline{S}_1^{(i)}) + S_{44}^{(i)} \sin(\phi_3^{(i)})(\underline{R}_f^{(i)T}\, \underline{a}_{12}^{(i)})}{\|\underline{R}_f^{(i)}\|^2} \tag{6.17}$$

A unique value of $\theta_2^{(i)}$ can therefore be determined from the above two equations.

The set of joint angles required for a single finger is now complete. For the hand the set of joint angles is completed by adapting the above procedure to each finger, i=1, 2, and 3.

6-2-4 Forward Displacement Analysis of the Hand

In the forward analysis the joint angles are given and it is necessary to find the position and orientation of the grasped object. This is accomplished by determining the position of each fingertip and the direction cosines of all the $\underline{S}_j^{(i)}$ (j = 1, 2, 3, and 4, and i=1, 2, and 3) are needed.

Figure 6.3 shows a skeletal model of a finger. The coordinate system x_i-y_i-z_i is defined such that x_i is along $\underline{a}_{12}^{(i)}$ and z_i is along $\underline{S}_1^{(i)}$. The fixed coordinate system x-y-z can be obtained from the coordinate system x_i-y_i-z_i, which is fixed on the i^{th} finger, by a rotation of angle $-\phi_1^{(i)}$ about axis z. The corresponding rotation matrix $M^{(i)}$ is in the form

$$M^{(i)} = \begin{bmatrix} \cos(\phi_3^{(i)}) & \sin(\phi_1^{(i)}) & 0 \\ -\sin(\phi_1^{(i)}) & \cos(\phi_1^{(i)}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6.18}$$

The components of the vector $\underline{R}^{(i)}$ in local coordinates are given by

$$\underline{R}^{(i)} = a_{23}^{(i)} \, \underline{a}_{23}^{(i)} + S_{44}^{(i)} \, \underline{S}_4^{(i)} \tag{6.19}$$

where

$$\underline{a}_{23}^{(i)} = \begin{bmatrix} c_2^{(i)} \\ 0 \\ s_2^{(i)} \end{bmatrix}_{x_i - y_i - z_i} \tag{6.20}$$

$$\underline{S}_4^{(i)} = \begin{bmatrix} s_{2+3}^{(i)} \\ 0 \\ c_{2+3}^{(i)} \end{bmatrix}_{x_i - y_i - z_i} \qquad \Big(c_{2+3} = \cos(\theta_2 + \theta_3) \;\; \text{and} \;\; s_{2+3} = \sin(\theta_2 + \theta_3)\Big) \tag{6.21}$$

$$\underline{R}^{(i)} = \begin{bmatrix} a_{23}^{(i)} c_2^{(i)} + S_{44}^{(i)} s_{2+3}^{(i)} \\ 0 \\ a_{23}^{(i)} c_2^{(i)} + S_{44}^{(i)} s_{2+3}^{(i)} \end{bmatrix}_{x_i - y_i - z_i} \tag{6.22}$$

The vector $\underline{R}_f^{(i)}$ in the fixed coordinates is given by

$$\underline{R}_f^{(i)} = M^{(i)}\, \underline{R}^{(i)} \tag{6.23}$$

In local coordinates,

$$\underline{S}_1^{(i)} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{x_i - y_i - z_i} \tag{6.24}$$

$$\underline{S}_2^{(i)} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}_{x_i - y_i - z_i} \tag{6.25}$$

$$\underline{S}_3^{(i)} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}_{x_i - y_i - z_i} \tag{6.26}$$

and the vector $\underline{S}_4^{(i)}$ is given by Equation 6.20. In the fixed coordinates these vectors can be expressed in the form

$$\underline{S}_{1f}^{(i)} = M^{(i)}\, \underline{S}_1^{(i)} \tag{6.27}$$

$$\underline{S}_{2f}^{(i)} = M^{(i)}\, \underline{S}_2^{(i)} \tag{6.28}$$

$$\underline{S}_{3f}^{(i)} = M^{(i)}\, \underline{S}_3^{(i)} \tag{6.29}$$

$$\underline{S}_{4f}^{(i)} = M^{(i)}\, \underline{S}_4^{(i)} \tag{6.30}$$

This completes the forward analysis of the i^{th} finger. The same procedure is applied to each finger to yield the forward analysis of the hand.

6-2-5 Force Analysis

Chapter 3 contained an extensive analysis of the contacting forces. In this section we will just show the method of computation involved in this process.

The characteristic matrix equation is given by Equation (3.2) and rewritten here as

$$J^T \Delta \underline{\$} = K \underline{\Omega} \tag{6.31}$$

where

$$J = [\ \underline{W}_1^{(1)}\ \underline{W}_2^{(1)}\ \underline{W}_3^{(1)} \ldots \underline{W}_1^{(3)}\ \underline{W}_2^{(3)}\ \underline{W}_3^{(3)}\ ] \tag{6.32}$$

$$K = \begin{bmatrix} K^{(1)} & 0 & 0 & 0 \\ 0 & K^{(2)} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & K^{(M)} \end{bmatrix} \tag{6.33}$$

$$K^{(i)} = \begin{bmatrix} \underline{\$}_{r1}^{(i)T} \Delta \underline{\$}_{1}^{(i)} & \cdots & \underline{\$}_{r1}^{(i)T} \Delta \underline{\$}_{N}^{(i)} \\ \underline{\$}_{r3}^{(i)T} \Delta \underline{\$}_{1}^{(i)} & \cdots & \underline{\$}_{r3}^{(i)T} \Delta \underline{\$}_{N}^{(i)} \end{bmatrix} \tag{6.34}$$

$$\underline{\Omega} = \begin{bmatrix} \omega_1^{(1)} \\ \omega_2^{(1)} \\ \vdots \\ \omega_3^{(M)} \end{bmatrix} \tag{6.35}$$

$$\Delta = \begin{bmatrix} 0 & I_3 \\ I_3 & 0 \end{bmatrix} \tag{6.36}$$

and where

J is a 6×9	dual matrix
Δ is a 6×6	matrix given by equation (3.7)
K is a 9×9	matrix
$\underline{\Omega}$ is a 9×1	vector
$\underline{\$}$ is a 6×1	dual vector.

The external wrench $\underline{W}$ on an object grasped by three fingers can be expressed in the following form:

$$\underline{W} = c_1^{(1)}\underline{W}_1^{(1)} + c_2^{(1)}\underline{W}_2^{(1)} + \dots + c_3^{(3)}\underline{W}_3^{(3)} \tag{6.37}$$

where $\underline{W}_1^{(i)}$, $\underline{W}_2^{(i)}$, $\underline{W}_3^{(i)}$ are three linearly independent contact wrenches with intensities $c_1^{(i)}$, $c_2^{(i)}$, and $c_3^{(i)}$ respectively due to the contact of the i^{th} finger. For a point contact with friction these contact wrenches are simply three forces. $\underline{W}_1^{(i)}$ will denote the force normal to the plane of contact whilst $\underline{W}_2^{(i)}$ and $\underline{W}_3^{(i)}$ are in the plane of contact. However, for a soft contact the contact wrenches consist of three forces together with a couple which is normal to the contact plane (see Salisbury [1982]). In a matrix form it can be written as follows:

$$\underline{W} = J\underline{c} \tag{6.38}$$

where

$$\underline{c} = \begin{bmatrix} c_1^{(1)} \\ c_2^{(1)} \\ \vdots \\ c_3^{(3)} \end{bmatrix} \tag{6.39}$$

and J is given above.

Solving Equation 6.37 for $\underline{c}$, using the MSVD of J (see Appendix B) yields the following explicit expression for $\underline{c}$

$$\underline{c} = V_1 B_1^{-1} U^T \Delta \underline{W} + V_2 \underline{\alpha} \tag{6.40}$$

where

$$\underline{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \tag{6.41}$$

is an arbitrary vector which will be determined using the optimization procedure presented in section 3.4.3.

- Determine the matrix J

Given the normal to the contact surface at the fingertip $P^{(i)}$, $\underline{n}^{(i)}$, the components of the wrench $\underline{W}_1^{(i)}$ can be computed as follows

$$\underline{W}_1^{(i)} = \begin{bmatrix} \underline{n}^{(i)} \\ \underline{R}_f^{(i)} \times \underline{n}^{(i)} \end{bmatrix} \tag{6.42}$$

where $\underline{R}_f^{(i)}$ is given by Equation 6.22.

$\underline{W}_2^{(i)}$ and $\underline{W}_3^{(i)}$ are the tangential wrenches. Let $\underline{t}_2^{(i)}$ and $\underline{t}_3^{(i)}$ represent any two perpendicular vectors in the plane of contact then

$$\underline{W}_2^{(i)} = \begin{bmatrix} \underline{t}_2^{(i)} \\ \underline{R}_f^{(i)} \times \underline{t}_2^{(i)} \end{bmatrix} \tag{6.43}$$

$$\underline{W}_3^{(i)} = \begin{bmatrix} \underline{t}_3^{(i)} \\ \underline{R}_f^{(i)} \times \underline{t}_3^{(i)} \end{bmatrix} \tag{6.44}$$

- Determine the matrix K

The screws $\underline{\$}_1^{(i)}$, $\underline{\$}_2^{(i)}$, and $\underline{\$}_3^{(i)}$ are also needed.

$$\underline{\$}_1^{(i)} = \begin{bmatrix} \underline{S}_1^{(i)} \\ \\ \underline{0} \end{bmatrix}_{x_i\text{-}y_i\text{-}z_i} \tag{6.45}$$

$$\underline{\$}_2^{(i)} = \begin{bmatrix} \underline{S}_2^{(i)} \\ \\ \underline{0} \end{bmatrix}_{x_i\text{-}y_i\text{-}z_i} \tag{6.46}$$

$$\underline{\$}_3^{(i)} = \begin{bmatrix} \underline{S}_3^{(i)} \\ \\ a_{23}^{(i)}\ \underline{a}_{23}^{(i)} \times \underline{S}_3^{(i)} \end{bmatrix}_{x_i\text{-}y_i\text{-}z_i} \tag{6.47}$$

These 6x1 vectors can be transformed to the fixed coordinate system using the following 6x6 rotation matrix

$$A^{(i)} = \begin{bmatrix} M^{(i)} & 0 \\ 0 & M^{(i)} \end{bmatrix}$$

where $M^{(i)}$ is given by Equation 6.18.

6.3 Path Generation

The tools developed in the previous section will be used here to generate the necessary information for the hand to follow a prespecified path. Given the initial and

final positions, trajectory planning generates a sequence of "control set points" for the control of the hand from the initial location to its destination.

Quite frequently, there exists a number of possible trajectories between the two given end points. For example, one may want to move the grasped object along a straight line path that connects the end-points (straight line trajectory); or to move the grasped object along a smooth curve that satisfies the position and orientation constraints at both ends (joint interpolated trajectory). In this work we will discuss only the straight line trajectory.

Straight Line Trajectory

It is required to move the grasped object along a straight line path between two known points specified by P_i and P_f in time T. The initial and final position and orientation of the grasped object are defined by

$$\text{initial} \begin{cases} P_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} & (6.48) \\ OR_i = \begin{bmatrix} \underline{i}_i & \underline{j}_i & \underline{k}_i \end{bmatrix} & (6.49) \end{cases}$$

$$\text{final} \left\{ \begin{array}{l} \mathrm{P_f} = \begin{bmatrix} x_f \\ y_f \\ z_f \end{bmatrix} \qquad (6.50) \\ \\ \mathrm{OR_f} = \begin{bmatrix} \underline{i}_f & \underline{j}_f & \underline{k}_f \end{bmatrix} \qquad (6.51) \end{array} \right.$$

The number of intermediate steps required between the initial and final states is based on the total change of position and orientation. A stepsize of 1.0 cm is chosen for the translation, and a 5 degree stepsize is chosen for the rotation. The number of intermediate steps is the greatest of the number given by the translation and the number given by the rotation

$$\text{nsteps} = \max\left(\frac{\Delta(\text{position})}{1.0}, \frac{\Delta(\text{orientation})}{5.0}\right)$$

A detailed analysis of this procedure can be found in Crane [1987].

A computer program was written to generate the required joint angles at the intermediate postions. These joint angles serve as input to the graphics program so that the rectilinear motion of the grasped object can be viewed on the screen. The next section will describe this graphics software as well as the hardware used for the animation.

6.4 Computer Graphics Simulation

The graphics animation was performed on the Silicon Graphics IRIS 4D/70 model workstation. The most important feature of this workstation is its capability of performing all the coordinate transformation in hardware. A set of custom VLSI chips

called the Geometry Engine are capable of transforming points, vectors and polygons, given in world coordinates, in screen coordinates at a rate of 69,000 3-D foating point coordinates per second.

To avoid flickering, during animation, a double buffer animation technique is used. This is done by using half of the available IRIS bitplanes for drawing while displaying the picture already drawn on the other half of the bitplanes. A swapbuffers command is performed, which gives a smooth animation free of flickering.

In drawing the hand model, two other features of the IRIS where used, namely backface polygon removal and the z-buffer. The backface polygon removal algorithm uses the normal to the surface to determine its visibility. If the normal has a component along the negative z-axis (into the screen), the surface is not displayed. The z-buffer is a separate depth buffer used to store the z-coordinate or depth of every pixel in the screen coordinate system. In use, the depth or z value of a new pixel to be written to the frame buffer is compared to the depth of that pixel already stored in the z-buffer. If the comparison indicates that the new pixel is in front of the pixel stored in the frame buffer, then the new pixel is written to the frame buffer and the z-buffer is updated with the new z value. If not, no action is taken. The simplicity of the algorithm is its greatest advantage. Quite importantly, it handles the hidden surfaces problem and the display of complex surface intersections trivially.

The combination of these two algorithms, backface polygon removal and z-buffer, leads to a very simple yet fast graphics programs, since most of the sorting is done in the hardware of the machine.

Figure 6.9 shows a solid model of the hand. Each finger has three joints, the joint angle values are displayed on the screen at all times allowing the user to check for joint

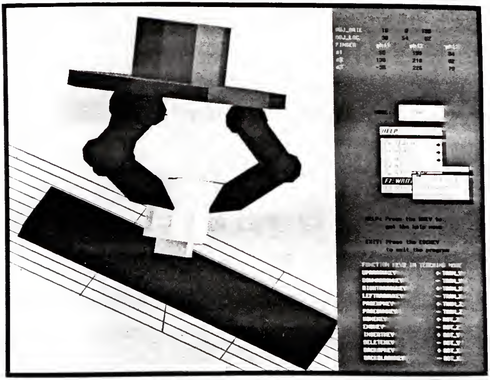


Figure 6.9

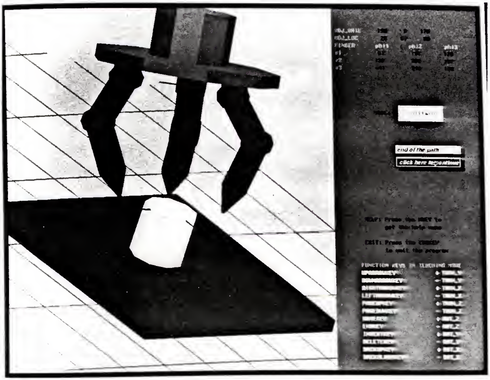


Figure 6.10

limits. Two modes of control are available: manual mode and autonomous mode.

In the manual mode, the user can modify the values of the joint angles using the mouse as input device, or he can guide the grasped object through a specified path using the keyboard. The described path can then be recorded and previewed by the user. In the autonomous mode the user is asked to input the following data:

- location of the object ($\underline{r}_{obj}$)
- orientation of the object (OR_{obj})
- location of the contact points in local coordinates ($\underline{r}^{(i)}$)
- normals at the contact points ($\underline{n}^{(i)}$).

The program computes the following data

- the required joint angles for each finger ($\phi_j^{(i)}$),
- the location of the palm,

and displays the solid model of the hand and the grasped object on the screen. The user can use the mouse to change the view point and zoom in and out.

The second step is path planning. The user specifies the location and orientation of the grasped object at the initial and target position. The program then computes the joint angles in the intermediate positions and the motion of the grasped object is previewed on the screen. This allows the user to evaluate the performance of the hand in executing the specified task. Figure 6.10 shows the solid model of the hand and the grasped object.

If the user is not satisfied with the generated path, he can change it by puting the program in a teaching mode. The user can then guide the object along the chosen steps and store the new path for later replay.

CHAPTER 7
CONCLUSION

This dissertation has been concerned with some kinestatic problems of the hand, namely the manipulating ability, the special configuration, and the optimal grasp. The first problem was to determine the characteristic matrix equation of the hand using the theory of reciprocal screws. A modified singular value decomposition was introduced to analyze the dual matrices of the characteristic matrix equation. This method, which is based on the theory of reciprocal screws assures a complete invariance of the results under a change of origin. Also, a novel method for analyzing the dual matrix, for the case of point contact with friction, is developed. This method significantly reduces the number of computations. Software was also developed to model a three-fingered hand. A straight line path, for the grasped object, was also generated between two positions specified by the user. This path can be previewed on the screen and modified if the user wishes to do so.

7.1 Future Work

The point contact with friction is a very attractive type of contact that leads to a substantial reduction of computations. However, it is not possible to have a perfect point contact between two rigid bodies. This is due, mainly, to the infinite pressure generated at the local contact, which damages the two contacting bodies. Figure 7.1 shows an example of a design of point contact. The pad allows a contact surface with most

objects, and the spherical joint assures the transmission of only pure forces between the finger and the object. This design can still be modeled by a point contact with friction where the point is located at the center of the wrist. In future work other designs should be investigated to preserve the same kinematic properties as the point contact and at the same time be feasable.

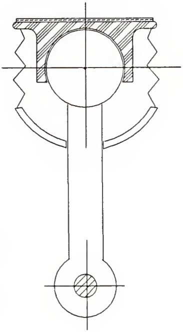


Figure 7.1

Computer graphics is a powerful tool that can help the simulation of different designs before actually building them. The simulation of the effect of gravity field on the grasped object will yield very realistic results. This area of research is still in its early stages as far as multi-fingered hands are concerned. Investigations of the workspace of mechanical hands could also be simplified if it is simulated using computer graphics.

APPENDIX A
THE SINGULAR VALUE DECOMPOSITION (SVD)

For the sake of completeness a summary of the method of the SVD (see Steven [1980]) is given here prior to developing a modified SVD for dual matrices in Appendix B. In this section we will assume throughout that A is an $m \times n$ matrix with rank(A)=r and m>n. (This assumption is made for convenience only; all the results will also hold if m<n). The method involves factoring A into a product VBU^T, where V is an $m \times m$ orthogonal matrix, U is an $n \times n$ orthogonal matrix, and B is an $m \times n$ matrix whose off diagonal entries are all 0's and whose diagonal elements satisfy

$$b_1 \geq b_2 \geq \dots \geq b_r > 0 \tag{A-1}$$

$$B = \begin{bmatrix} b_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & b_r & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{A-2}$$

$$B = \begin{bmatrix} B_1 & 0 \\ 0 & 0 \end{bmatrix} \tag{A-3}$$

B_1 is an $r \times r$ diagonal matrix and the b_i's determined by this factorization are unique and are called the singular values of A. The factorization VBU^T is called the singular value decomposition (SVD) of A.

Let $U = [U_1 \ \ U_2]$ where U_1 is an $n \times r$ matrix and $V = [V_1 \ \ V_2]$ where V_1 is an $m \times r$ matrix.

Consider the equation

$$A \underline{x} = \underline{b} \tag{A-4}$$

where $\underline{x}$ is an $n \times 1$ vector and $\underline{b}$ is an $m \times 1$ vector.

Since $A = VBU^T$, $AU = VB$ and

$$[AU_1 \ \ AU_2] = [V_1 B_1 \ \ 0]. \tag{A-5}$$

Hence Eq.(A-4), which can be written as

$$BU^T \underline{x} = V^T \underline{b} \tag{A-6}$$

or

$$B_1 U_1^T \underline{x} = V_1^T \underline{b} \tag{A-7}$$

has the general solution

$$\underline{x} = U_1 B_1^{-1} V_1^T \underline{b} + U_2 \underline{Y} \tag{A-8}$$

$$\underline{0} = V_2^T \underline{b} \tag{A-9}$$

where $\underline{Y}$ is an arbitrary vector of dimension $(n\text{-}r)\times 1$.

APPENDIX B
THE MODIFIED SINGULAR VALUE DECOMPOSITION (MSVD)

The procedure developed in Appendix A cannot be applied to dual matrices such as J, because this transformation is based on the formation of the matrix J^TJ which involves the orthogonal product of screws.

Lipkin & Duffy [1984] showed that the orthogonal product of screws is non-invariant with a translation of origin or a change of scale. However, the reciprocal product of screws is invariant under any projective transformation of 3 dimension space and leads always to meaningful results. This concept can be used to modify the classical singular value decomposition, by forming the following matrix,

$$A = J^T \Delta J \tag{B-1}$$

where

$$J = [\ \underline{\$}_{r1}^{(1)}\ \underline{\$}_{r2}^{(1)}\ \underline{\$}_{r3}^{(1)} \dots \underline{\$}_{r1}^{(M)}\ \underline{\$}_{r2}^{(M)}\ \underline{\$}_{r3}^{(M)}\ ] \tag{B-2}$$

and

$$\Delta = \begin{bmatrix} 0 & I_3 \\ I_3 & 0 \end{bmatrix} \tag{B-3}$$

$$A = \begin{bmatrix} 2\underline{S}_1^T\underline{S}_{o1} & \underline{S}_1^T\underline{S}_{o2}+S_1^T\underline{S}_{o2} & \cdots \\ S_1^T\underline{S}_{o2}+S_1^T\underline{S}_{o2} & 2S_2^T\underline{S}_{o2} & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix} \tag{B-4}$$

A is invariant with a translation of origin. Further it is symmetric and therefore diagonalizable. Hence a 9×9 orthogonal matrix V and a 9×9 diagonal matrix B can be found such that

$$A = V^T\, BV \tag{B-5}$$

$$B = \begin{bmatrix} b_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & b_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{B-6}$$

$$B = \begin{bmatrix} B_1 & 0 \\ 0 & 0 \end{bmatrix} \tag{B-7}$$

Finally defining U' by

$$U' = J\,V = [\,U \quad 0\,] \tag{B-8}$$

where U' is a 6x9 dual matrix and U is a 6x6 dual matrix, then

$$U = J\,V_1 \tag{B-9}$$

and

$$0 = J\,V_2 \tag{B-10}$$

The modified singular value decomposition of the dual matrix J can thus be expressed in the form

$$J = U'V^T = [U \quad 0]\begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} = U\,V_1^T \tag{B-11}$$

For the case where J is singular i.e. rank(J) = r < 6 the above analysis yields

$$J = [U_1 \mid U_2\,]V^T \tag{B-12}$$

where

$$U_2^T\,\Delta U_2 = 0 \tag{B-13}$$

and

$$U_1^T \Delta U_1 = B_1 \tag{B-14}$$

the rank of U_2 is equal to 6-r

Solution of the Caracteristic Matrix Equation

The characteristic matrix equation of the hand (see Equation (2.2)) is

$$J^T \Delta \underline{\$} = K\underline{\Omega} \tag{B-15}$$

Replacing J using (B-11), yields

$$V_1 U^T \Delta \underline{\$} = K\underline{\Omega} \tag{B-16}$$

and therefore (see B-9 and B-10)

$$U^T \Delta \underline{\$} = V_1^T K\underline{\Omega} \tag{B-17}$$

and

$$\underline{0} = V_2^T \underline{\Omega} \tag{B-18}$$

It can be shown that the $\{\underline{U}_i\}$ form a basis of the space of all possible motions. Therefore it is always possible to find an X such that

$$\underline{\$} = U\underline{X} \tag{B-19}$$

Substituting (B-19) in (B-17) yields

$$U^T \Delta U\underline{X} = V_1^T K\underline{\Omega} \tag{B-20}$$

From (B-9)

$$U^T \Delta\ U = V_1^T\ J^T \Delta\ J\ V_1 \tag{B-21}$$

and from (B-5)

$$U^T \Delta\ U = B_1 \tag{B-22}$$

Substituting (B-20) into (B-14) and solving for $\underline{X}$ yields

$$\underline{X} = B_1^{-1}\ V_1^T\ K\underline{\Omega} \tag{B-23}$$

Finally the solution of the characteristic matrix equation (B-9) can be expressed by

$$\underline{\$} = U\ B_1^{-1}\ V_1^T\ K\underline{\Omega} \tag{B-24}$$

together with

$$\underline{0} = V_2^T\ K\underline{\Omega} \tag{B-25}$$

Grasping with Point Contact with Friction

In the case of a point contact with friction between the finger and the object, three forces can be transmitted. These forces can be represented by three unit line vectors

$$\underline{\$}_1 = \begin{bmatrix} \underline{S}_1 \\ \underline{S}_{o1} \end{bmatrix} \qquad \underline{\$}_2 = \begin{bmatrix} \underline{S}_2 \\ \underline{S}_{o2} \end{bmatrix} \qquad \underline{\$}_3 = \begin{bmatrix} \underline{S}_3 \\ \underline{S}_{o3} \end{bmatrix} \tag{B-26}$$

Where $\{ \underline{S}_1 , \underline{S}_2 , \underline{S}_3 \}$ is an orthonormal basis (ONB) of $\mathbb{R}^3$.

and $\underline{S}_{o1} = \underline{r}_1 \times \underline{S}_1$; $\underline{S}_{o2} = \underline{r}_1 \times \underline{S}_2$; $\underline{S}_{o3} = \underline{r}_1 \times \underline{S}_3$.

Let $R_1 = [\underline{r}_1 \times]$ a 3x3 skew-symmetric matrix representing the cross product with the vector $\underline{r}_1$ which is a vector from the origin to the point contact. The vector $\underline{S}_{oi}$ becomes

$\underline{S}_{oi} = R_1 \, \underline{S}_i$; $\quad i = 1, 2, 3.$

For another point of contact on the same object, three forces can be transmitted also

$$\underline{\$}'_i = \begin{bmatrix} \underline{S}'_i \\ \underline{S}'_{oi} \end{bmatrix}; \quad i=1, 2, 3, \tag{B-27}$$

where $\{ \underline{S}'_1 , \underline{S}'_2 , \underline{S}'_3 \}$ can be taken as an ONB of $\mathbb{R}^3$ thus a rotation matrix A_2 can be found such that

$$\underline{S}'_i = A_2 \, \underline{S}_i \, ; \qquad i = 1, 2, 3. \tag{B-28}$$

$$\underline{S}'_{oi} = \underline{r}_2 \times \underline{S}'_i = \underline{r}_2 \times A_2 \underline{S}_i = R_2 A_2 \underline{S}_i \; ; \quad i = 1, 2, 3. \tag{B-29}$$

For the case of three points contacting an object the matrix J is

$$J = \begin{bmatrix} S_1 & S_2 & \cdots & A_3 \underline{S}_3 \\ R_1 S_1 & R_1 S_2 & \cdots & R_3 A_3 \underline{S}_3 \end{bmatrix} \tag{B-30}$$

The analysis of the matrix $A = J^T \Delta J$ can be simplified by using the following theorem (Steven [1980]):

Theorem: Let A an m×n matrix and B an n×m matrix (m<n), then rank(AB) = rank(BA) < m, and the set of eigenvalues is common to both matrices.

proof: Let $\underline{V}$ an eigenvector of the matrix AB corresponding to the eigenvalue λ, we can write

$$AB\underline{V} = \lambda\underline{V}. \tag{B-31}$$

Multiplying by B yields

$$B(AB)\underline{V} = B(\lambda\underline{V}) \quad ==> \quad (BA)(BV) = \lambda(BV) \tag{B-32}$$

which shows that the matrix BA has the same eigenvalue λ as AB and the corresponding eigenvector is $B\underline{V}$ (QED).

By virtue of this theorem the matrix $J^T \Delta J$ and $(\Delta J)J^T$ have the same set of eigenvalues thus the same characteristic equation.

Since the matrices A_i 's are rotation matrices then $A_i^T = A_i^{-1}$.

The set of vectors { $\underline{S}_1$, $\underline{S}_2$, $\underline{S}_3$ } is an ONB then

$$I_3 = \underline{S}_1\, \underline{S}_1^T + \underline{S}_2\, \underline{S}_2^T + \underline{S}_3\, \underline{S}_3^T. \tag{B-33}$$

The matrix JJ^T can be expressed in the following form

$$JJ^T = \begin{bmatrix} 3I_3 & R_1^T+R_2^T+R_3^T \\ R_1+R_2+R_3 & R_1^TR_1+R_2^TR_2+R_3^TR_3 \end{bmatrix} \tag{B-34}$$

and since the matrices R_i's are skew-symmetric then $R_i^T = -R_i$ which yields

$$JJ^T = \begin{bmatrix} 3I_3 & -R_1-R_2-R_3 \\ R_1+R_2+R_3 & -R_1R_1-R_2R_2-R_3R_3 \end{bmatrix} \tag{B-35}$$

$$\Delta JJ^T = \begin{bmatrix} R_1+R_2+R_3 & -(R_1^2+R_2^2+R_3^2) \\ 3I_3 & -(R_1+R_2+R_3) \end{bmatrix} \tag{B-36}$$

Let $$\underline{\Upsilon} = \begin{bmatrix} \underline{v}' \\ \underline{v} \end{bmatrix} \tag{B-37}$$

where $\underline{v}$ and $\underline{v}'$ are 3x1 vectors, be an eigenvector of ΔJJ^T corresponding to the eigenvalue λ,

$$\Delta JJ^T\,\underline{\Upsilon} = \lambda\underline{\Upsilon} \tag{B-38}$$

and using (B-36)

$$(R_1 + R_2 + R_3)\,\underline{v}' - (R_1^2 + R_2^2 + R_3^2)\,\underline{v} = \lambda\underline{v}' \tag{B-39}$$

$$3\,\underline{v}' - (R_1 + R_2 + R_3)\underline{v} = \lambda \underline{v} \tag{B-40}$$

Eliminating the vector $\underline{v}'$ between these two equations yields,

$$[3(R_1^2 + R_2^2 + R_3^2) - (R_1 + R_2 + R_3)^2 + \lambda^2 I_3]\underline{v} = \underline{0} \tag{B-41}$$

and the characteristic equation can be expressed in the form

$$\mathrm{DET}[3(R_1^2 + R_2^2 + R_3^2) - (R_1 + R_2 + R_3)^2 + \lambda^2 I_3] = 0 \tag{B-42}$$

This equation shows that the eigenvalues of the matrix $J^T \Delta J$ which are the the same as those for $\Delta J J^T$ are three pairs of form ($+\lambda_j$,$-\lambda_j$), j=1, 2, 3. Further, the singularities are independent of the rotation matrices A_i's and are dependent solely upon the R_1 , R_2 , and R_3 , which represent the locations of the contact points on the grasped object.

This result can be generalized for n contact points, the characteristic equation for which is

$$\mathrm{DET}[n(R_1^2 + \cdots + R_n^2) - (R_1 + \cdots + R_n)^2 + \lambda^2 I_3] = 0 \tag{B-43}$$

Equation (B-41) yields three orthonormal vectors $\underline{v}_i$ corresponding to the three eigenvalues b_i^2, i=1,2,3 respectively. The vectors $\underline{v}_i'$, $\underline{v}_i''$ corresponding to $\pm b_i$ are obtained from Equation (B-40)

$$\underline{v}_i' = R\underline{v}_i + \frac{b_i \underline{v}_i}{3} \tag{B-44}$$

$$\underline{v}_i'' = R\underline{v}_i - \frac{b_i\underline{v}_i}{3} \qquad \text{(B-45)}$$

where $$R = \frac{1}{3}(R_1 + R_2 + R_3). \qquad \text{(B-46)}$$

Using Equations (B-5) and (B-9) yields the relations between U, V_1, and Υ

$$U = p\Delta\Upsilon \qquad \text{(B-47)}$$

$$V_1 = pJ^T\Upsilon B_1^{-1} \qquad \text{(B-48)}$$

where $$p = \sqrt{\frac{3}{2}} \quad \text{(for the case of three fingers)} \qquad \text{(B-49)}$$

and $$B_1 = \mathrm{diag}(b_1, -b_1, b_2, -b_2, b_3, -b_3). \qquad \text{(B-50)}$$

Invariance of the Characteristic Equation

Here the case of three contact points is considered and a proof by induction can be formulated for the case of n contact points. A translation of the origin from O to O′ transforms the R_i matrices as follow

$$R_i' = R_i + V \qquad i = 1, 2, 3 \qquad \text{(B-51)}$$

where $V = [\underline{v}\times]$, and $\underline{v} = \underline{OO'}$.

The new characteristic equation of the system assumes the form

$$DET[3(R_1'^2 + R_2'^2 + R_3'^2) - (R_1' + R_2' + R_3')^2 + \lambda^2 I_3] = 0 \tag{B-52}$$

where

$$R_i'^2 = R_i^2 + VR_i + R_i V + V^2 \qquad i = 1, 2, 3. \tag{B-53}$$

and

$$(R_1+R_2+R_3+3V)^2=(R_1+R_2+R_3)^2+3V(R_1+R_2+R_3)+3(R_1+R_2+R_3)V+9V^2 \tag{B-54}$$

Substituting (B-53) and (B-54) in (B-52) yields (B-42) which demonstrates the invariance with respect to a change of origin.

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BIOGRAPHICAL SKETCH

The author was born in Sousse, Tunisia on a Saturday August 15, 1959. After graduating from high school with honors, he joined the National School for Engineers, where he got the diploma of "Ingenieur Principal" in June 1984. The author joined the Ph.D. program at the University of Florida in the Fall of 1984.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Joseph Duffy, Chairman
Graduate Research Professor of
Mechanical Engineering

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George N. Sandor, co-Chairman
Research Professor of
Mechanical Engineering

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Carl D. Crane III
Assistant Professor of
Mechanical Engineering

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Ralph G. Selfridge
Professor of
Computer and Information Sciences

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

John Staudhammer

John Staudhammer
Professor of
Electrical Engineering

This dissertation was submitted to the Graduate Faculty of the College of Engineering and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August 1989

W J Shafer

Dean, College of Engineering

Dean, Graduate School